Open Space Policies and Urban Spatial Structure

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Revised: October 4, 2001

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Abstract

There is widespread public support for open space provision and for efforts by government to limit sprawl. We demonstrate that open space policies should not be viewed as independent of—or necessarily compatible with—growth management goals. We examine the impacts of open space designation on the urban landscape in a spatial city model with two important and empirically-relevant features: (1) residents prefer to live close to open space and (2) open space amenities attract migrants to the city. Our main findings are that open space designation can produce leapfrog development when it is located outside of the city; the effect of open space on the total area of developed land in the city is ambiguous; the location, size and configuration of open space can all affect development densities throughout the city. Our analysis identifies the key factors that determine the impacts of open space and yields insights into the design of effective land-use policies.

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I. INTRODUCTION

There is strong public support for the provision of open space in urbanizing areas. According to the Land Trust Alliance, 454 open space initiatives were voted on in local and state elections in the U.S. between 1998 and 2000. Eighty-four percent of these measures were approved, providing over $17 billion for land purchases. Americans also express strong support for anti-sprawl policies. For example, a poll commissioned by Smart Growth America in 2000 found that 78 percent of Americans support efforts by government to curb sprawl.

The purpose of this paper is to investigate the impact of open space designation on the spatial structure of urban areas. Hedonic analyses of residential housing prices consistently reveal an inverse relationship between housing prices and distance to urban environmental amenities (e.g., [1], [18], [20], [26], [28], [31]).[1] Likewise, a consistent finding of the migration literature is that natural amenities positively influence migration decisions (e.g., [10], [15], [19], [22], [24], [28], [31]). That people find it more desirable to live close to open space suggests that open space can influence the defining features of the urban landscape, such as the location and density of residential development and the overall size of the city. As such, open space policies should not be viewed as independent of—or necessarily compatible with—efforts to limit sprawl. While the provision of open space removes some land from the path of the bulldozers, it may create incentives for even more land development.[2]

[1] Contingent valuation studies (e.g., [5], [14], [21]) also reveal positive willingness to pay for the preservation of urban open space.
[2] The belief that open space provision is an effective means of limiting sprawl appears to be widespread. For example, a stated goal of the American Farmland Trust is to "identify … productive farmland threatened by sprawl … (and) then work with communities to plan and effect farmland conservation." In their 1999 annual report, the Land Trust Alliance proclaims, "We will always stand ready to help every land trust … to save the very face of America."

We examine the effects of open space on the urban landscape with an open city model developed in the Alonso-Muth-Mills tradition: (a) residents of the city face tradeoffs between commuting and housing costs, (b) land developers choose the location and density of development to maximize expected profits, (c) the city's population is endogenous and adjusts through in-migration and out-migration, and (d) land and housing prices are determined in a spatial market equilibrium. In contrast to traditional models, we relax the assumption of a featureless plain and allow residential sites to be differentiated by the level of open space amenities, which enters as an argument in the utility functions of residents. The implications of the model are explored analytically and through a series of simulations.

We consider a land conservation group that purchases or obtains conservation easements on undeveloped land. The open space provides use values to the city's residents in the form of recreational opportunities, and the utility derived from the open space depends on the distance to and the size of the parcel. We show that open space may cause discontinuous (leapfrog) development when placed outside the city, as land near the open space is developed for residential housing. We also examine an equilibrium in which the city has expanded to encompass the open space. We show that open space provision can increase or decrease the total amount of development in the city. The net effect of open space on city size depends on factors such as the size of the parcel, the relationship between size and amenity levels, and transportation costs. Finally, we consider how the shape of open space parcels (e.g., continguous parcels, greenbelts, amenity corridors) affects the spatial structure of the city. We show that the form of open space can have different effects on development densities throughout the city.

Numerous studies use spatial city models to explain observed characteristics of the urban landscape (a recent review is found in Anas et al. [2]). Several analyses use this framework to examine causes of urban sprawl. Brueckner [6, 7] defines urban sprawl as excessive spatial expansion and identifies the economic forces that drive the expansion. Brueckner, however, does not explain spatial patterns of urban expansion such as fragmented leapfrog development. Mills [23] uses a monocentric-city model to show that sprawl can arise from landowner decisions to preserve a ring of undeveloped land for future use. Related studies on urban sprawl include Fujita and Kashiwadani [15], who find in an empirical analysis that urban sprawl may be one type of efficient land-use pattern, and Ogawa and Fujita [32], who use a two-dimensional urban land use model to examine equilibrium urban configurations. Our approach is unique in its consideration of the effect of open space policies on urban spatial structure. Perhaps the closest studies to ours are Brueckner *et al*. [8], who consider the effect of amenities on the location of different income groups within cities, and Wu [33], who analyzes the effect of major geographical features (e.g., oceans) on development patterns. Our analysis is also closely related to several empirical studies that model decisions to develop individual land parcels as a function of environmental amenities (e.g., [13], [17]).

In Section II, we develop the model of the urban landscape and characterize the spatial market equilibrium. In Section III, we use the model to analyze the impacts of open space policies on urban spatial structure. Section IV presents discussion and conclusions.

II. AN OPEN CITY MODEL WITH ENVIRONMENTAL AMENITIES

We first outline the structure of our model. Our model has three basic components. First, we examine the household location decision. Households have preferences defined over residential space, a non-housing good, and environmental amenities and face a budget constraint that includes the costs of commuting to a central business district (CBD). Solution of the household utility maximization problem characterizes the demand for housing at each location. Second, we consider the land development decision. Developers choose the location and density of development to maximize expected profits. The solution to the profit maximization problem characterizes the supply of housing at each location. Third, we examine a spatial market equilibrium in which, among other conditions, demand for and supply of housing are equated.

The Household Location Decision

The household decision model conforms to some of the basic assumptions of the Alonso-Muth-Mills model, including a central business district (CBD), a population of households with identical incomes and preferences, and commuting costs that depend on the distance between residences and the CBD. The landscape is represented by a Cartesian coordinate plane R^2, with the CBD located at the origin (0,0) and the *u*- and *v*-axis representing west-east and north-south directions, respectively. The distance between the CBD and a residential site at (*u*,*v*) is $x \equiv \left(u^2 + v^2\right)^{0.5}$, which together with the angle of incidence, θ, determines the location of the household, where $(u,v) = (x\cos\theta, x\sin\theta)$. However, in contrast to the traditional model, we relax the assumption that the landscape is a featureless plain and allow residential sites to be differentiated by the level of environmental amenities. The spatial heterogeneity of amenities over the landscape is represented by a distribution function $a(u,v)$ defined over the plane.

Each household takes the spatial distribution of amenities, $a(u,v)$, and the rental price of housing (e.g., rental price per square foot of residential space), $p(u,v)$, as given. However, by selecting a residential location (*u*,*v*), households are also choosing a level of amenities and a rental price. More formally, each household chooses a combination of residential space (*q*), location (*u*,*v*), and a numeraire non-housing good (*g*) to maximize utility $U(q, g, a(u,v))$ subject to the budget constraint $p(u,v)q + g + tx = y$, where y is gross household income, and t is the round-trip commuting cost per mile. We use a Cobb-Douglas specification of the utility function: $U(q,g,a(u,v)) = q^{\alpha} g^{1-\alpha} a(u,v)^{\gamma}$, where α and γ are positive parameters.[3]

The first-order conditions for the utility maximization problem can be combined to yield the differential equation system

(1) $$\frac{\alpha}{p(u,v)} \frac{\partial p(u,v)}{\partial u} = \frac{\gamma}{a(u,v)} \frac{\partial a(u,v)}{\partial u} - \frac{t}{(y-tx)} \frac{\partial x}{\partial u},$$

(2) $$\frac{\alpha}{p(u,v)} \frac{\partial p(u,v)}{\partial v} = \frac{\gamma}{a(u,v)} \frac{\partial a(u,v)}{\partial v} - \frac{t}{(y-tx)} \frac{\partial x}{\partial v}.$$

Solving the system, we obtain the bid price at each location

(3) $$p^{*}(u,v) = p_0 \left[\frac{a(u,v)^{\gamma}(y-tx)}{y} \right]^{\frac{1}{\alpha}},$$

where p_0 is the rental price and the environmental amenities at the CBD, $a(0,0)$, is normalized to 1. Equation (3) is a household's maximum willingness to pay to locate at (u,v). It can be viewed as an equilibrium condition, in the sense that when (3) holds, the level of household utility is uniform across the landscape and households have no incentive to move. This can be verified by substituting the optimal levels of residential space $q^{*}(u,v) = \alpha(y-tx) / p^{*}(u,v)$ and the non-housing good $g^{*}(u,v) = (1-\alpha)(y-tx)$ into the household's utility function to obtain the indirect utility function

(4) $$V = \alpha^{\alpha}(1-\alpha)^{1-\alpha} y / p_0^{\alpha},$$

which shows that household utility is independent of location.

[3] The Cobb-Douglas specification has been widely used in spatial city models for analytical tractability (see, for e.g., [3], [27], [37]).

The bid price function (3) reveals the difference between our model and the traditional monocentric city model. In the standard case, the landscape is assumed to be a featureless plain (i.e., $a(u,v)=1$) and bid prices always fall with the distance from the CBD to compensate residents for the cost of commuting. However, with spatial variation in environmental amenities, the pattern of housing prices is more complicated. For example, if open space is located at the fringe of a metropolitan area, a household may be willing to pay more to live closer to it than to the CBD. This issue will be analyzed in detail in Section III.

The Land Development Decision

Land developers choose the density and location of development to maximize expected profits. As with many other investments, residential real estate development involves uncertainty over future returns, and land development can be viewed as irreversible since development decisions can only be undone at great expense. Irreversibility and uncertainty may give rise to an option value associated with delaying irreversible land development and receiving new information on the value of development (e.g., [9]). We account for the effects of uncertainty and irreversibility by modeling land development as a two-period decision process. Development in the first period is irreversible and second period profits from development are stochastic (for a similar application, see Arrow and Fisher [4]). This is a simplified version of the general infinite-time horizon model (e.g., [11], but one that still captures the essential features of the problem.

We assume that developers choose the location and density of development to maximize the present value of profits. Let $\pi(s,u,v)$ be the profit per acre from developing land at (u,v) in the current period, where s is development density (e.g., square feet of residential space per acre). $\pi(s,u,v)$ can be written $\pi(s,u,v) = p(u,v)s - c(s,u,v)$, where $p(u,v)$ is the rental price that developers take as given and $c(s,u,v) = r(u,v) + c(s)$ are total costs that include the site-specific land rent $r(u,v)$ (e.g., rental price for a one-acre lot) and material and labor costs $c(s)$. For simplicity, we assume $c(s) = c_0 + s^{\beta}$, where $\beta > 1$. Discounted second period profits are given by $\pi_2(s_2,s,u,v) = p(u,v)s_2 - c(s_2 - s,u,v)$, where the second period development density at (u,v) is s_2, and $s_2 \geq s$ implies that development is irreversible.[4] This formulation captures the cases in which the development density is increased on land that was previously developed ($s > 0$) and land that was previously undeveloped ($s = 0$).

We assume that developers know the current profits but not the second period profits until the beginning of the second period. Thus, from the perspective of the current period, $\pi_2(s_2,s,u,v)$ is uncertain. Expectations of $\pi_2(s_2,s,u,v)$ are assumed to be conditioned on current housing and land rents, $p(u,v)$ and $r(u,v)$. Specifically, $\pi_2(s_2,s,u,v) = hp(u,v)s_2 - c(s_2 - s,u,v)$ with probability w and $\pi_2(s_2,s,u,v) = lp(u,v)s_2 - c(s_2 - s,u,v)$ with probability (1-w), where $h > l > 0$. We assume that if housing prices are high in the second period, the development density will be increased ($s_2 > s$) and, if housing prices are low, the density will remain the same ($s_2 = s$).[5]

Our objective is to characterize the spatial market equilibrium in the *current* period. To this end, we solve for the developer's current bid price and optimal development density. However, our model recognizes that current decisions depend on future decisions because of the

[4] We suppress the discounting term to avoid notational clutter. Second period costs, $c(s_2 - s,u,v) = r(u,v) + c_0 + c(s_2 - s)^{\beta}$, are a function of $s_2 - s$ because material and labor costs are one-time expenditures. Land rental costs $r(u,v)$ are incurred every period.

irreversibility of development and uncertainty over future returns. Specifically, we write the developer's maximization problem as

(5) $$\max_{s,u,v}\ [p(u,v)s - c(s,u,v)] + w[hp(u,v)s_2^* - c(s_2^* - s,u,v)] + (1-w)lp(u,v)s\ ,$$

where s_2^* solves $hp(u,v) = \partial c(s_2 - s,u,v)/\partial s_2$. The first-order conditions are

(6) $$\theta p(u,v) - \frac{\partial c(s,u,v)}{\partial s} = 0,$$

(7) $$\left[\theta s + wh(s_2^* - s)\right]\frac{\partial p(u,v)}{\partial u} - \frac{\partial c(s,u,v)}{\partial u} - w\frac{\partial c(s_2^* - s,u,v)}{\partial u} = 0\ ,$$

(8) $$\left[\theta s + wh(s_2^* - s)\right]\frac{\partial p(u,v)}{\partial v} - \frac{\partial c(s,u,v)}{\partial v} - w\frac{\partial c(s_2^* - s,u,v)}{\partial v} = 0\ ,$$

where $\theta \equiv 1 + wh + (1-w)l$.[6]

Using (6), the expression for $c(s,u,v)$, and the condition for s_2^* in the high profit case, we derive the optimal density in the two stages

(9) $$s^* = \left(\frac{\theta}{\beta}\right)^{\frac{1}{\beta-1}} p(u,v)^{\frac{1}{\beta-1}},\quad s_2^* - s^* = \left(\frac{h}{\beta}\right)^{\frac{1}{\beta-1}} p(u,v)^{\frac{1}{\beta-1}}\ .$$

Substituting these results and the cost function into (7) and (8), we obtain the following differential equation system

(10) $$\left[\theta\left(\frac{\theta}{\beta}\right)^{\frac{1}{\beta-1}} + wh\left(\frac{h}{\beta}\right)^{\frac{1}{\beta-1}}\right] p(u,v)^{\frac{1}{\beta-1}} \frac{\partial p(u,v)}{\partial u} = (1+w)\frac{\partial r(u,v)}{\partial u}$$

(11) $$\left[\theta\left(\frac{\theta}{\beta}\right)^{\frac{1}{\beta-1}} + wh\left(\frac{h}{\beta}\right)^{\frac{1}{\beta-1}}\right] p(u,v)^{\frac{1}{\beta-1}} \frac{\partial p(u,v)}{\partial v} = (1+w)\frac{\partial r(u,v)}{\partial v}$$

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[5] In the low housing price case, we assume that *l* is sufficiently small and fixed costs c_0 are sufficiently high to make additional development unprofitable.

Solving the differential equation system, we obtain the land rental price

$$r(u,v)=\left[\frac{p^{**}(u,v)}{\phi}\right]^{\frac{\beta}{\beta-1}}+c, \tag{12}$$

where

$$\phi=\left[\frac{(1+w)\beta^{\beta/(\beta-1)}}{(\beta-1)\left(\theta^{\beta/(\beta-1)}+wh^{\beta/(\beta-1)}\right)}\right]^{\frac{\beta-1}{\beta}},$$

$p^{**}(u,v)$ is the developer's minimum selling price for residential housing at location (u,v), and c is the constant of integration determined by the condition that profits be zero in a competitive market equilibrium. Substituting (9) and (12) into the expression for profits in (5) and setting the resulting expression equal to zero yields $c=-c_0$. Rearranging (12) and substituting into (9) gives the optimal density in the current period

$$s^{*}(u,v)=\left(\frac{\theta\phi}{\beta}\right)^{\frac{1}{\beta-1}}[r(u,v)+c_0]^{\frac{1}{\beta}}. \tag{13}$$

Equation (13) shows that the development density increases with the land price.

Equilibrium Conditions

Having derived the behavioral functions for both households and developers, we now characterize spatial equilibrium for land development. Spatial equilibrium is achieved when 1) housing rental prices are bid up in desirable locations such that no household wants to move, 2) the price that households are willing to pay equals the price that developers are willing to accept at each location, 3) land rental prices are bid up in desired locations such that development profits are zero everywhere and developers are indifferent to the location of development, 4) the

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[6] To derive (6)-(8), we make use of the results $\partial s_2^{*}/\partial s=1$ and $hp(u,v)=\partial c(s_2^{*}-s,u,v)/\partial s_2^{*}$.

total demand for residential space equals the total supply, and 5) at the city boundary, the land rental price equals the exogenous agricultural rent r_{ag}.

The first equilibrium condition is satisfied when housing rental prices are given by (3), because the household's bid price function implies that household utility is independent of location (see equation 4). The second equilibrium condition is satisfied when

$$p^*(u,v) = p^{**}(u,v)\,. \qquad (14)$$

Using (3), (4), (12), and (14), we obtain a reduced-form expression for the equilibrium land rental price

$$r(u,v) = \left[\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\phi^{\alpha}V}\right]^{\frac{\beta}{\alpha(\beta-1)}} a(u,v)^{\frac{\gamma\beta}{\alpha(\beta-1)}}(y-tx)^{\frac{\beta}{\alpha(\beta-1)}} - c_0\,. \qquad (15)$$

The third condition requires profits from development to be independent of location. This condition is satisfied when housing and land prices are given by (3) and (15), respectively. This is verified by combining (9), (12), and (15) to produce expressions for equilibrium prices $p(u,v)$ and $r(u,v)$ and optimal densities s^* and s_2^*, and then substituting these into the expression for profits in (5).

To characterize the fourth equilibrium condition, note that household density, $n(u,v)$, equals the development density (floor space per acre) divided by housing demand per household (floor space per household): $n(u,v) = s^*(u,v)/q^*(u,v)$. Land will be developed if the developer's bid price for land is greater than the agricultural rent r_{ag}. Thus, the developed area can be represented by the set $D \equiv \{(u,v) | r^*(u,v) > r_{ag}\}$, and the fourth equilibrium condition is satisfied when

(16) $$\iint_{r^*(u,v)>r_{ag}} \frac{s^*(u,v)}{q^*(u,v)} dudv = N,$$

where *N* is the total number of households in the city. The last equilibrium condition is given by $r^*(\bar{u},\bar{v}) = r_{ag}$, where $(\bar{u},\bar{v})$ is a set of points at the city's boundary.

Equations (3), (15) and (16) completely characterize the spatial equilibrium. We consider an open city with perfect mobility. In this case, migration equalizes utility across cities in equilibrium and, thus, *V* is exogenous from the perspective of a single city. The number of households in the city (*N*) is endogenous and determined by (16).[7] Comparative statics illustrate the mechanics of the model. By (3) and (15), rising income and falling transportation costs cause land and housing prices to increase throughout the city. This implies an increase in the population of the city. Higher housing and land prices reduce the demand for land by residents (q^*), increase the density of housing (s^*), and expand the city boundary. In-migration restores the equilibrium condition (16). Rising agricultural rents cause the city to contract and, by (16), reduce the city's population. In equilibrium, land and housing prices are unaffected because out-migration reduces the aggregate demand for land. An increase in the exogenous utility level causes out-migration, which proceeds faster than the city shrinks, so that prices and development density fall and each household consumes more land and housing. One way to understand this result is to recognize that if utility goes up while income remains unchanged, prices must fall so that a higher utility level can be reached with given income.

III. THE EFFECT OF OPEN SPACE ON URBAN SPATIAL STRUCTURE

[7] In a closed city model, there is no migration and, thus, *N* is fixed. In this case, *V* is endogenous and determined by (16).

The equilibrium model of the city is used to examine the effects of open space policies on urban spatial structure. We assume that a conservation agency purchases (or acquires easements on) parcels of undeveloped land. Designation as open space results in two important changes in the status of the land. First, residents of the city are afforded access to the open space. For simplicity, we assume that residents do not have access to other undeveloped lands. Second, the agency increases the quality of the open space through investments such as tree planting and trail development. The designated open space provides amenities to residents in the form of recreational benefits. As found in the hedonic studies on property values cited above, the level of amenities at each residential location is a declining function of distance to the open space.

Comparison of the spatial market equilibria in baseline and open space scenarios identifies the effects of open space on the city structure. In the baseline scenario, there is no open space and no spatial variation in amenities. In this case, the city is circular and all land within the city boundary is developed. We examine two types of open space equilibria that are distinguished by the location of the open space relative to the city's boundary in the baseline scenario. In the first type of open space equilibrium, the agency has acquired land that lies outside of the city's baseline boundary. In this case, we investigate the potential for the open space to produce leapfrog (discontinuous) development. In the second type of open space equilibrium, the agency has acquired land that lies within the city's baseline boundary. In this case, we examine the effects of the open space on the total area of developed land. Lastly, we compare several open space equilibria to gauge the effects of open space configuration (e.g., parks, greenbelts) on development densities. We provide a combination of analytical results and simulations (parameter values are listed in Table 1). The purpose of the simulations is to illustrate some of the possible consequences of open space policies.

Open Space Location and the Potential for Leapfrog Development

We start with the case in which a single parcel of land located outside the city boundary has been designated as open space. In the baseline with no open space, environmental amenities are distributed uniformly across the landscape and normalized to one (i.e., $a(u,v)=1$). In this case, the developed area $D \equiv \{(u,v)|r^*(u,v) > r_{ag}\}$ is a circle because the land price depends on only the distance from the CBD (see equation 15). This is shown in panel (a) of Figure 1. The shading in Figure 1 shows the contour of the land price within the city. In panel (a), prices are highest (dark shading) at the CBD and decline as the distance to the CBD increases.

In the open space scenario, we assume that the parcel is small relative to the whole landscape and simply treat it as a point (below, we examine how the size and shape of open space parcels affect spatial patterns of development). Without loss of generality, the land designated for open space is assumed to be located at (d,0). The amenity distribution function is assumed to have the form $a(u,v)=1+a_d e^{-\eta z}$, where $z=\left((u-d)^2+v^2\right)^{0.5}$ is the distance between location (u,v) and the open space, a_d is the level of amenities provided to a household located at (d,0), and η is a parameter determining the rate at which amenities decline as one moves farther away from the open space. Because open space provides higher amenities to households who live closer to it, the designation of open space changes the relative attractiveness of each location. To see this, subsititute $a(u,v)=1+a_d e^{-\eta z}$ into (3) and note the negative relationship between z and the household's bid price.

In the simulation we set a_d=0.16 and $\eta=1$, and consider equilibria in which open space is placed at (2,0), (3,0), and (4,0). When the open space is at (3,0), for example, the amenity level at the CBD is 15% lower than it is next to the open space. Panels (b)-(d) of Figure 1 show the city boundary after the designation of open space. Designation of land for open space increases amenities in the city, which causes in-migration and expansion of the city in all directions. Open space also changes the relative attractiveness of different locations. Unless the open space is located at the CBD, the developed area is no longer circular because development is concentrated around the open space.

When the open space is located close enough to the city boundary (e.g., at (2,0) and (3,0)), the city encompasses the open space in equilibrium. Land prices are elevated in the area surrounding the open space but drop off quickly as one moves east and farther from the city center. When the open space is located at a sufficient distance from the city center, leapfrog development can occur. For example, when the open space is located at (4,0), there are two developed areas separated by agricultural land. Land in the agricultural median is too far from both the city center and the open space for prices to be bid up above the agricultural rent.

Given the location of open space at (*d*,0), a necessary and sufficient condition for open space to result in leapfrog development is that there be four solutions to

$$a\left(\left|d-u\right|\right)^{\gamma}\left(y-t\left|u\right|\right)=\frac{\phi^{\alpha}(r_{ag}+c_0)^{\alpha(\beta-1)/\beta}V}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}. \tag{17}$$

In words, this condition requires that the land price in (15) be equal to the agricultural rent r_{ag} at four points along the horizontal axis. When $r^*(u,0)=r_{ag}$, $r^*(u,v)<r_{ag}$ for the given value of u and any $v\neq 0$ because moving off the line (*u*,0) increases the distance to the city center and the open space. Thus, the vertical lines that pass through the solutions to (17) separate the developed area into disjoint sets. If there are four solutions to (17), then the developed area *D* can be divided into two disjoint subsets (e.g., panel (d) in Figure 1).

Alternatively, if the designation of open space causes leapfrog development, $r^*(u,0)$ must have a minimum in the interval u=[0, d] where $r^*(u,0) < r_{ag}$. At all extreme points in [0, d], the second derivative of the land price with respect to u is

(18) $$r^{*\prime\prime}(u,0) = \frac{\gamma\beta[r^*(u,0)-c_0]}{\alpha(\beta-1)a(z)^2}\left[a(z)a''(z)-(1+\gamma)a'(z)^2\right].$$

If the expression in (18) is negative for all $z \in [0,d]$, then the open space does not cause leapfrog development. This implies that

(19) $$\frac{d[a'(z)/a(z)]}{da(z)}\frac{a(z)}{[a'(z)/a(z)]} < \gamma \qquad \forall z \in [d,0],$$

where the left-hand side of (19) is the elasticity of the growth rate in amenities as one moves closer to the open space and the right-hand side is the elasticity of utility with respect to amenities. If the left-hand side is always less than the right-hand side, then the amenities from open space are not attractive enough to cause leapfrog development.

In sum, open space provision is shown to affect the spatial patterns of land development. In particular, when the open space is located far enough from the metropolitan area and provides sufficiently high amenities, it may cause leapfrog development. Open space may also affect the total area of developed land. When open space is assumed to be dimensionless, it always increases the total acreage of residential development in the city because the higher amenities from open space cause in-migration and greater demand for residential space by current residents. However, when the level of amenities depends on the area of open space, the designation of open space may increase or decrease the total developed area, depending on the relationship between the area of land preserved for open space and the area of increased development. This issue is explored next.

Open Space Size and the Impact on Total Developed Area

To determine the effect of open space designation on total developed area, we must evaluate the relationship between the area of land protected from development and the area of additional land developed in response to the designation of open space. In contrast to the analysis in the previous section, the level of amenities is assumed to depend on the size of the open space parcel. Specifically, the amenity distribution function is assumed to take the form $a(u,v) = a(z,S)$, where S is the area of open space, and z is the distance from (u,v) to the geographic center of the open space at $(d,0)$. In this section, we assume that the shape of the open space parcel does not affect the amenity level (in the next section, we explore the effect of open space configurations on urban spatial structure). We consider open space equilibria in which the designated land lies within the city's baseline boundary and evaluate the tradeoff between the area of land protected from development and the area of land developed in response to increased amenities.

In the baseline with no open space, the total developed area is

$$A_0 = \iint_{D_0} du dv = \pi x^2, \tag{20}$$

where D_0 is a circle with diameter x defined by

$$\left[\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\phi^{\alpha} V}\right]^{\frac{\beta}{\alpha(\beta-1)}} \left(y - tx\right)^{\frac{\beta}{\alpha(\beta-1)}} - c_0 = r_{ag}. \tag{21}$$

Solving for x from (21) and substituting it into (20), we obtain

$$A_0 = \frac{\pi}{t^2}\left(y - \lambda\right)^2, \tag{22}$$

where $\lambda = \left[\phi^{\alpha}(r_{ag} + c_0)^{\alpha(\beta-1)/\beta} V\right] \Big/ \left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right]$. When open space is located within the city boundary, the total developed area after the designation of open space equals

(23) $$A_1 = \iint_{D_1} du dv - S,$$

where D_1 is the developed area defined by $D_1 = \left\{(u,v) \mid a(z,S)^{\gamma}\left(y - tx\right) > \lambda\right\}$.

If $A_1 > A_0$, preserving land for open space increases the total area of developed land. This may occur when a small area of open space provides a high level of amenities that leads to in-migration and greater demand for residential space. On the other hand, if a large parcel of open space provides a relatively low level of amenities, then preserving land for open space can reduce total land development ($A_1 < A_0$). This can be shown analytically for a special case in which the open space provides the same level of amenities to all households regardless of location, and the amenity function takes the form $a(S) = 1 + (e^{\delta S} - 1)$, where 1 is the base level of amenities, $(e^{\delta S} - 1)$ is the increase in amenities provided by the open space, and δ is the elasticity of amenities with respect to the area of open space.[8]

With this amenity function, the total developed area with open space equals

(24) $$A_1 = \frac{\pi}{t^2}\left(y - \lambda e^{-\delta S}\right)^2 - S.$$

A comparison of (22) and (24) establishes that $A_1 > A_0$ if and only if

(25) $$\delta > \frac{1}{S}\left[\ln \lambda - \ln\left(y - \sqrt{(y-\lambda)^2 - t^2 S/\pi}\right)\right].$$

The expression (25) indicates that preserving land for open space is more likely to increase the total developed area if the amenity level is sensitive to the area of open space. Since the right-hand side of (25) approaches zero as *t* approaches zero, condition (25) will hold for a given δ

[8] This specification assumes increasing returns to scale in amenity provision. This implies, for instance, that the amenities from two separate open space parcels is lower than the amenities from a single parcel of equal size.

when transportation costs fall sufficiently. The reason for this result is that low transportation costs magnify the effects of the open space amenities on in-migration and associated land development.

The size of the open space parcel also affects the likelihood that open space increases the total developed area. Equation (24) shows that the total developed area depends on open space area *S*. The function has two interior extrema, given by

$$S_{1,2} = \frac{1}{\delta} \ln \left(\frac{4\delta\lambda^2}{2y\delta\lambda \pm \sqrt{4y^2\delta^2\lambda^2 - 8\lambda\delta t^2/\pi}} \right),$$

where $S_1 < S_2$. By checking the second-order condition, we can show that S_1 is a minimum and S_2 is a maximum. Figure 2 provides one possible depiction of (24). Open space provision initially decreases the total developed area relative to baseline development A_0.[9] With more open space, total developed area increases relative to A_0 and then decreases again. Figure 2 reveals that under some conditions open space can increase or decrease the total developed area. In the case illustrated here, total development declines initially because the amenities provided by the open space are too low to encourage much additional land development. A large amount of open space also reduces total development because it reduces the land available for development at locations relatively close to the CBD. When the total area of open space is between $\underline{S}$ and $\overline{S}$, open space actually causes more development.

Figure 3 shows the simulated effect of open space under the more general assumption that amenities depend on both the area of open space and the distance from the open space. The

[9] Recall from the previous section that a dimensionless open space parcel always increases total developed area. In the present case, a parcel with zero area provides no amenities and so $A_1 = A_0$ at *S*=0.

amenity distribution function has the form $a(z,S) = 1 + a_d(e^{\delta S} - 1)e^{-\eta z}$, which implies that the amenity level is increasing in the area of open space *S* and declining in the distance to the open space *z*. In panels (a) and (b), the amenity level is relatively responsive to the open space area ($\delta = 0.75$). In these cases, open space (hatched circles) expands the city boundary (the outer circles) enough so that the total developed land area increases. In panels (c) and (d), δ is smaller (0.10) and the provision of open space decreases the total developed area.

Open Space Configuration and the Impact on Development Densities

Open space can take many forms. Area-featured open space includes community parks and preserved agricultural or forest lands. Line-featured open space includes greenbelts, riverfront parks, and wildlife corridors. To demonstrate the effects of open space configuration on urban spatial structure, we compare the results of simulations with three common types of open space: a park, a greenbelt, and a corridor. The three forms of open space have the same acres (0.25π), but the park is a circle located at (2,0): $\{(u,v) \mid (u-2)^2 + v^2 \le 0.5^2\}$; the greenbelt is a ring around the CBD: $\{(u,v) \mid 1.968 \le u^2 + v^2 \le 2.031\}$; and the corridor is a belt that passes through (2,0): $\{(u,v) \mid 1.9345 \le u \le 2.0654, -3 \le v \le 3\}$. The amenity distribution function takes the form $a(z,w) = 1 + a_d(e^{\mu w} - 1)e^{-\eta z}$, where *w* is the diameter of the park or the width of the greenbelt or corridor, *z* is the distance to the center of the park or the closest distance to the middle of the greenbelt or corridor, and (a_d, μ, η) are parameters whose values are given in Table 1.

Figure 4 shows the city structure with the park, greenbelt and corridor, where the darker shading signifies higher development density. Figure 4 shows that the open space configuration has implications for the shape of the city as well as development density throughout the city.

Panel (a) shows the circular city in the baseline with no open space. When a park is placed at (2,0), the city takes on an oblong shape, as development is concentrated around the park boundaries (panel b). The park draws some development away from the city center and western portions of the city, and results in new new development east of the park. With the greenbelt, in contrast, the city retains its circular shape and the development density is still highest in the city center (panel c).[10] Note the potential for development to take place outside of the greenbelt. This is less likely to occur if the width of the greenbelt is increased, which increases commuting costs to the CBD. In panel (d), development is concentrated along the corridor, and some new development takes place on the eastern edge of the corridor. In sum, the cases presented here suggest that open space parcels that are symmetric with respect to the CBD (the greenbelt compared to the park and corridor) have less impact on the city structure. In addition, there tend to be larger zones of developed land around more concentrated open space parcels (the park compared to the greenbelt and corridor) because amenity levels remain high as one moves away from the parcel.

IV. Discussion and Conclusions

There is widespread public support for efforts to increase the amount of open space and to limit sprawl patterns of land development (e.g., low-density, discontinuous development). There also appears to be widespread belief that these objectives are compatible—that the provision of open space is an effective means of limiting sprawl. The central purpose of this paper is to formally examine the relationship between open space policies and urban spatial structure. To this end, we develop a spatial city model with two empirically-relevant features: (1) people prefer to live

[10] When the greenbelt provides a sufficiently high level of amenities, the development density near the greenbelt can be higher than in the inner rings. However, the city retains its circular

close to open space in order to derive use values from recreation, and (2) open space amenities increase migration to a city. These features of our model are supported, respectively, by many hedonic analyses of housing prices and studies of the determinants of migration. Our analysis shows that these features have important implications for the impacts of open space on urban spatial structure.

We identified two potential effects of designating open space outside of the city boundary. First, we show that the city may expand to encompass the open space. This is more likely to happen if the open space is closer to the city boundary and if it provides a high level of amenities. A second possibility is that leapfrog development takes place. Leapfrog development is more likely to occur if the parcel is farther from the city. In this case, residents are willing to endure a long commute to work only if they can live very close to the open space. Land in the agricultural median is too far from the city and the open space to be attractive for development. This development pattern is evident around a number of major U.S. cities. For example, people choose to live in amenity-oriented communities like Boulder, Colorado and Marin County, California, even if this requires a commute into the city for employment.

Despite the fact that open space provision restricts development on some land, it's effect on the total area of developed land is ambiguous. Open space may increase migration to a city, giving rise to incentives for additional land development. As well, depending on the placement and configuration of the open space parcel, development may occur in areas that, otherwise, would have remained undeveloped. The ultimate impact on total development depends on factors such as the size of the open space parcel, the relationship between size and amenity levels, and transportation costs. For instance, if an open space parcel is sufficiently large, it can reduce total development by eliminating desirable sites for development near the city center.

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shape as long as the greenbelt is centered around the CBD.

While open space is clearly just one factor that affects migration and land development, it is not difficult to think of high-growth cities whose desirability is closely linked to open space provision. Portland, Oregon, and Seattle, Washington are prominent examples.

Our results have implications for policies designed to control sprawl patterns of development. First, open space provision should not be viewed, in general, as an effective tool for controlling sprawl. Open space may be a cause rather than a cure, and attention needs to be given to how open space alters incentives for land development. Second, open space may be effective in limiting land development if used in concert with other growth controls. For example, if the total area of developable land is restricted by an urban growth boundary (or a geographical feature such as a mountain range), then open space provision within the boundary achieves a reduction in developed area relative to the case in which all land within the boundary is developed. Finally, the configuration of open space can affect the density of land development throughout the city. In some cases, open space can help to concentrate development close to the city center, but in other cases it can draw development out of the urban core.

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TABLE 1
The Values of Parameters Used in the Simulations

Parameter	Value	Interpretation
α	1/3	Households spend one-third of income after commuting costs on housing
γ	0.5	The elasticity of utility with respect to amenities.
y	$40,000	Gross household income
t	$1,000	Annual commuting cost per mile (roundtrip)
r_{ag}	$700	Agricultural land rent per acre
β	4/3	The ratio of housing value to non-land construction costs
c_0	$300	Fixed development costs per acre
w	0.5	The probability of an increase in future profit from residential development
l	0.5	Future profit from residential development will be down by 50%
h	1.5	Future profit from residential development will be up by 50%
a_d	0.16	Amenity distribution function parameter
η	1	Amenity distribution function parameter
V	3700	The utility level

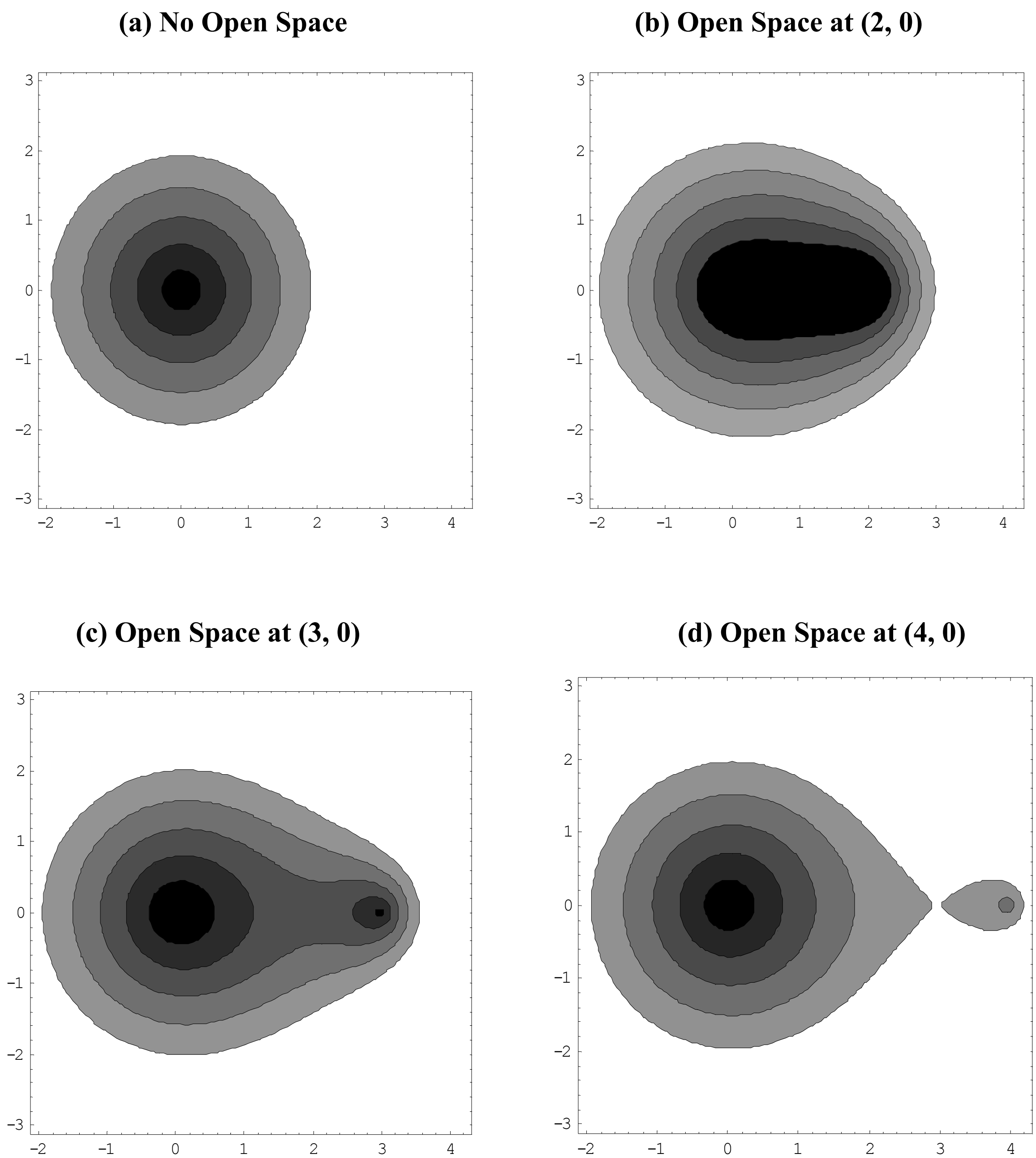

FIG. 1. The impact of open space locations on urban spatial structure and land price

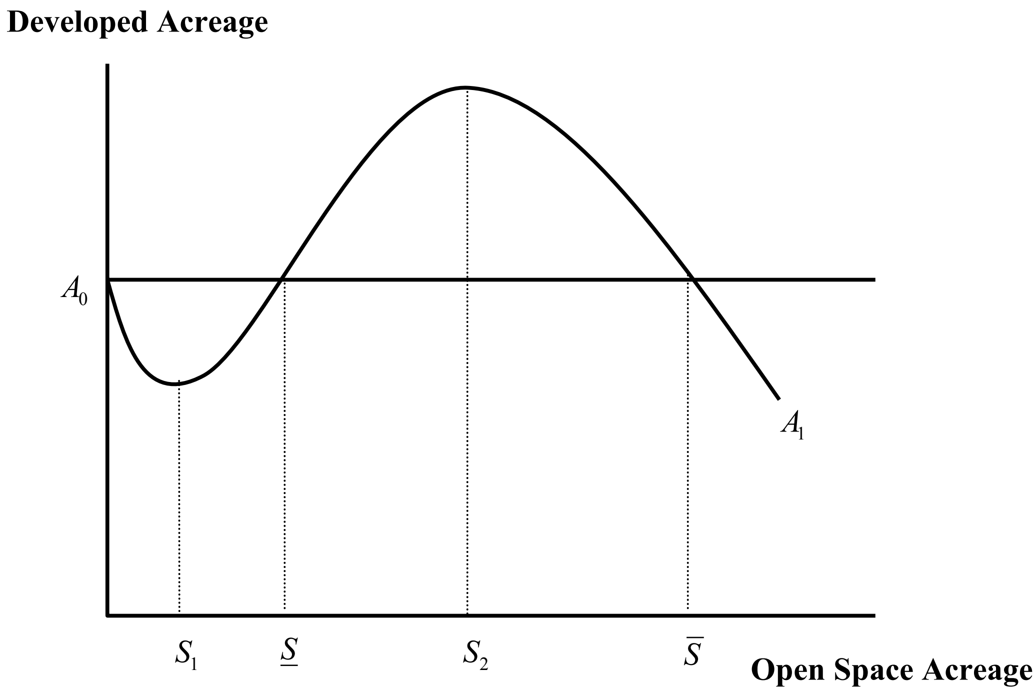

FIG. 2. The effect of open space area on total developed area

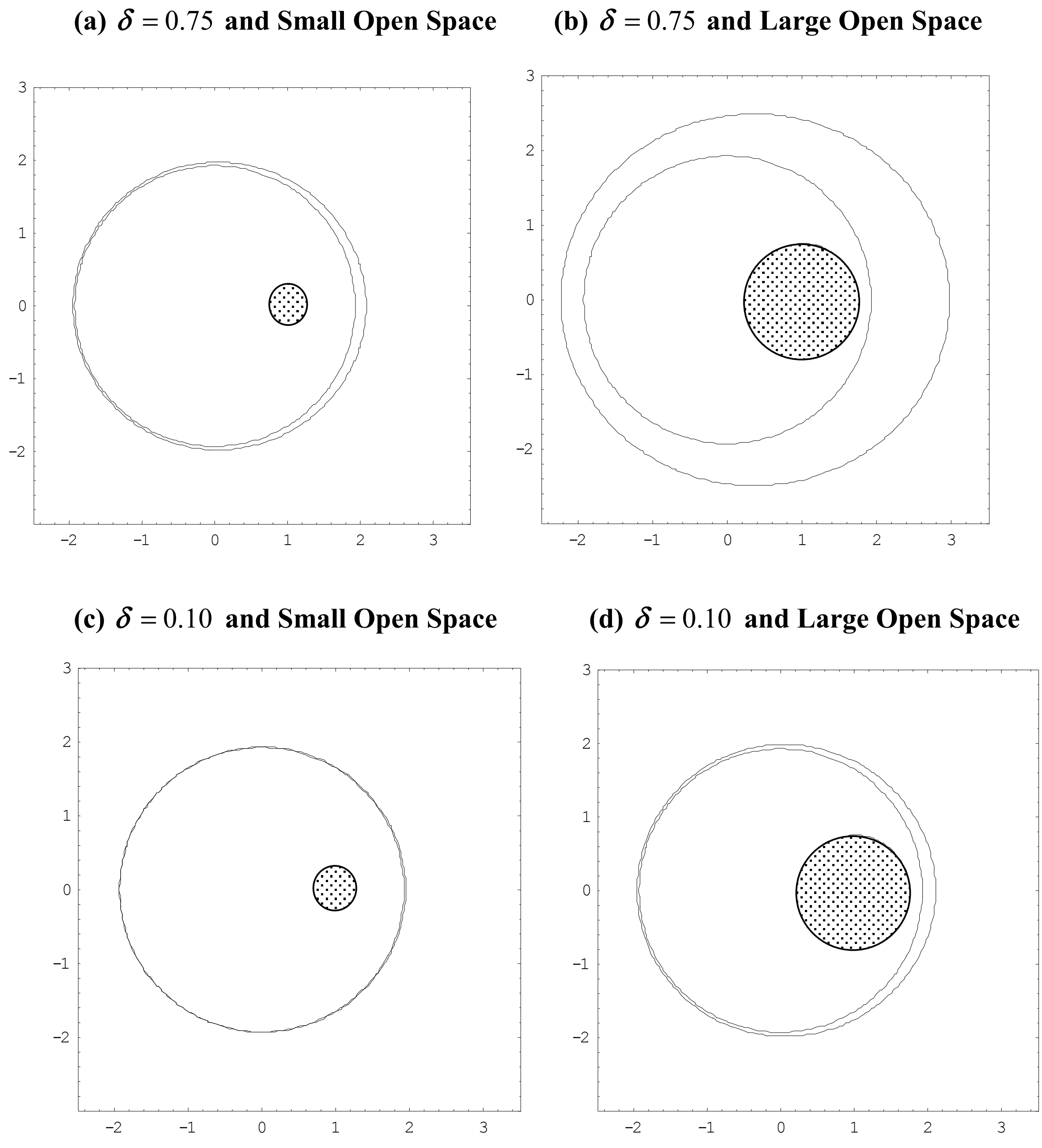

FIG. 3. The effect of open space area and amenities on total developed area

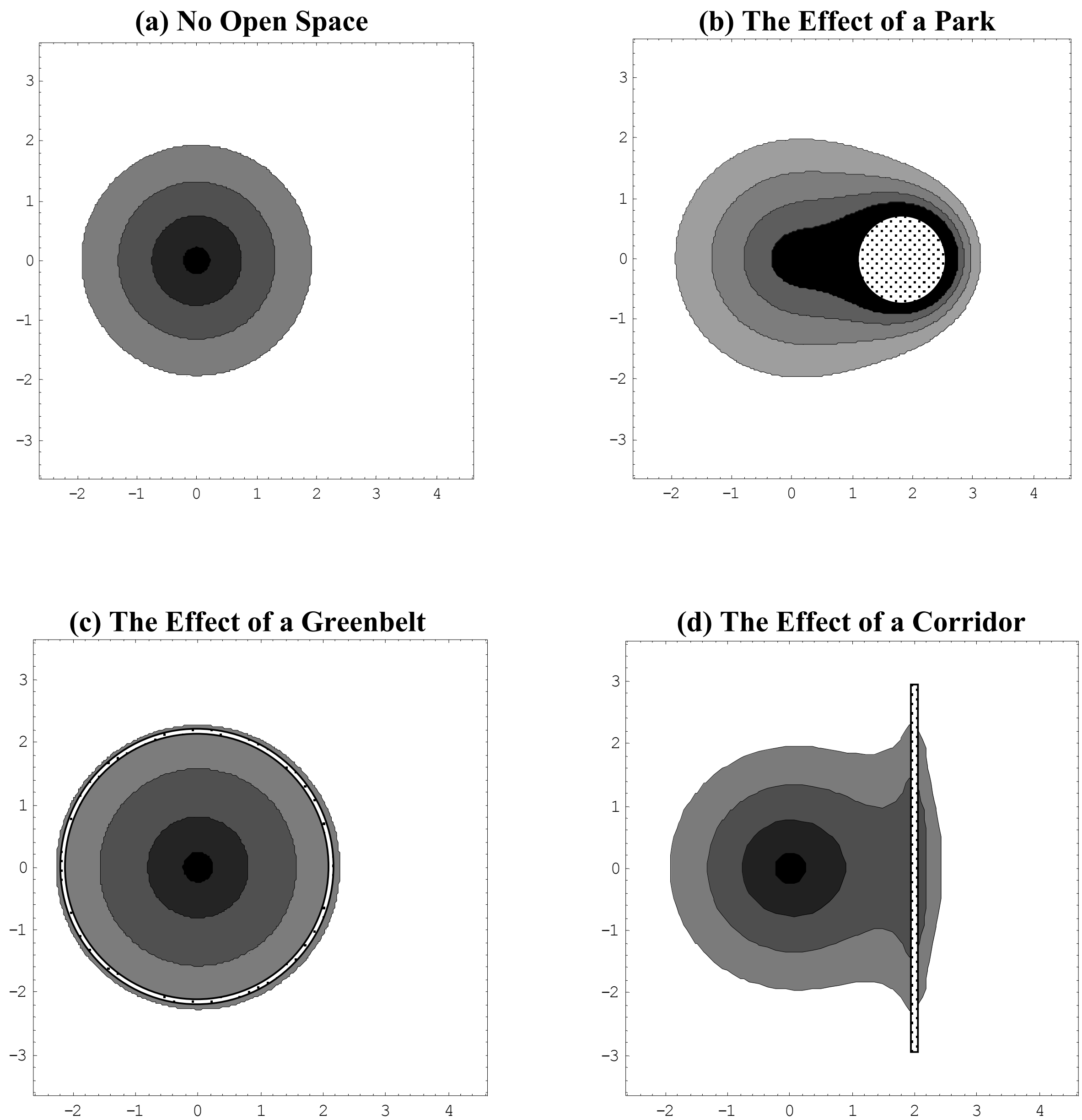

FIG. 4. The effect of open space configuration on urban spatial structure and development density