Economics of Agglomeration

Cities, Industrial Location, and Regional Growth

MASAHISA FUJITA
Kyoto University

JACQUES-FRANÇOIS THISSE
Université catholique de Louvain

CAMBRIDGE
UNIVERSITY PRESS

of industries in the world economy. Consequently, as argued by Venables (1996), dealing with the intermediate sector allows us to explain the possible emergence of a core–periphery structure at the international level. However, instead of following Venables (who assumes that both the intermediate and final sectors operate under increasing returns and monopolistic competition), we use a simpler framework. As in Section 4.2.1, we assume that the intermediate sector produces a differentiated good and exhibits increasing returns; however, the final sector produces a homogeneous good and exhibits constant returns. Finally, it is supposed that workers remain in their region. We then show that both sectors concentrate within the same region provided that the transport costs of the intermediate goods are sufficiently high (typically when they are nontradable). This is so even when the transport cost of the final good is very low. Indeed, the agglomeration of the intermediate sector firms makes it profitable for the final sector firms to agglomerate with them despite the wage gap generated by the immobility of workers.

In summary, the core–periphery structure may emerge owing to the migration of workers and the imperfectly competitive nature of the final sector or to the existence of an imperfectly competitive intermediate sector when workers are immobile. This result is very important for the space-economy, and thus it is crucial to know how it depends on the specificities of the framework employed. First, the use of the CES utility and iceberg cost leads to a convenient setting in which demands have a constant elasticity. However, such a result conflicts with research in spatial pricing theory in which demand elasticity varies with distance. Second, although the iceberg cost is able to capture the fact that shipping is resource-consuming, such a modeling strategy implies that any increase in the mill price is accompanied with a proportional increase in transport cost, which often seems unrealistic.[6] Finally, although models are based on very specific assumptions, they are often beyond the reach of analytical resolution, forcing authors to appeal to numerical investigations.[7] As recognized by Krugman (1998, 164) himself, "To date, the new economic geography has depended heavily on the tricks summarized in Fujita, Krugman, and Venables (1999) with the slogan 'Dixit–Stiglitz, iceberg, evolution and the computer.' "

This state of affairs has led Ottaviano and Thisse (1998) and Ottaviano, Tabuchi, and Thisse (2002) to revisit the core–periphery model using an alternative framework that involves downward-sloping linear demands and a linear transport cost measured in terms of the numéraire. Such a setting, which is very popular in location theory (Beckmann and Thisse 1986; Greenhut, Norman, and Hung 1987), takes us far away from the model used by Krugman and offers the advantage of yielding analytical solutions. Although the conclusions are not exactly the same as those derived by Krugman, this alternative model also yields a core–periphery structure once transportation costs are sufficiently low. Therefore, the core–periphery structure seems to be robust against very different formulations of preferences and transport technologies.

The linear model permits the study of different spatial price policies (Ottaviano and Thisse 1998). Because mill pricing yields the same qualitative results as in Section 9.2, we restrict ourselves to the case of discriminatory pricing in Section 9.4. This framework is very simple to use and is also suitable for studying the welfare properties of the core–periphery structure – an issue that has been untouched in most economic geography models. In Section 9.5, using the model of Section 9.4, we focus on the interplay between history and expectations in the formation of the economic space when migrants maximize the intertemporal value of their utility flows.

The work of Krugman has triggered a plethora of contributions, which have been surveyed by Ottaviano and Puga (1998). As noted earlier, the main result obtained by Krugman is the monotone relationship between the degree of agglomeration and the transportation cost level. In Section 9.6, the generality of such a relationship is discussed through several modifications of the basic model.

9.2 THE CORE–PERIPHERY MODEL

Although we focus on a two-region economy in this chapter, it will prove convenient to have a more general framework for subsequent developments.

9.2.1 The Framework

The economic space is made of R regions. The economy has two sectors: the modern sector ($\mathbb{M}$) and the traditional sector ($\mathbb{T}$). There are two production factors: the high-skilled workers and the low-skilled workers. The $\mathbb{M}$-sector produces a continuum of varieties of a horizontally differentiated product under increasing returns using skilled labor as the only input. The $\mathbb{T}$-sector produces a homogeneous good under constant returns using unskilled labor as the only input.

The economy is endowed with L unskilled workers and with H skilled workers. The skilled workers are perfectly mobile between regions, whereas the unskilled are immobile. As discussed in Section 8.2, this extreme assumption is partially justified because the skilled are more mobile than the unskilled. The share of unskilled workers in region r is fixed and denoted $0 \leq \nu_r \leq 1$ for $r = 1, \ldots, R$. The share of skilled workers in each region r is variable and denoted by $0 \leq \lambda_r \leq 1$ for $r = 1, \ldots, R$.

Although both consumption and production take place in a specific region, it is notationally convenient to describe preferences and technologies without explicitly referring to any particular region.

Preferences are identical across all workers and described by a Cobb–Douglas utility,

$$U = Q^{\mu}T^{1-\mu}/\mu^{\mu}(1-\mu)^{1-\mu} \qquad 0 < \mu < 1, \tag{9.1}$$

where Q stands for an index of the consumption of the modern sector varieties, and T is the consumption of the output of the traditional sector. When the modern sector provides a continuum of varieties of size M, the index Q is given by

$$Q = \left[\int_0^M q(i)^\rho di\right]^{1/\rho} \qquad 0 < \rho < 1, \tag{9.2}$$

where $q(i)$ represents the consumption of variety $i \in [0, M]$. Hence, each consumer displays a preference for variety. In (9.2), the parameter ρ stands for the inverse of the intensity of desire for variety over the differentiated product. When ρ is close to 1, varieties are close to perfect substitutes; when ρ decreases, the desire to spread consumption over all varieties increases. If we set

$$\sigma \equiv \frac{1}{1-\rho},$$

then σ is the elasticity of substitution between any two varieties, which varies between 1 and ∞. Because there is a continuum of firms, each firm is negligible, and the interactions between any two firms are zero, but aggregate market conditions (e.g., the average price across firms) affect each firm. This provides a setting in which firms are not competitive (in the classic economic sense of having infinite demand elasticity), but at the same time they have no strategic interactions with one another (see (9.4) below).[8]

If Y denotes the consumer income, $p^{\mathbb{T}}$ the price of the traditional good, and $p(i)$ the price of variety i, then the demand functions are

$$T = (1-\mu)Y/p^{\mathbb{T}} \tag{9.3}$$

$$q(i) = \frac{\mu Y}{p(i)} \frac{p(i)^{-(\sigma-1)}}{P^{-(\sigma-1)}} = \mu Y p(i)^{-\sigma} P^{\sigma-1} \qquad i \in [0, M], \tag{9.4}$$

where P is the price index of the differentiated product given by

$$P \equiv \left[\int_0^M p(i)^{-(\sigma-1)} di\right]^{-1/(\sigma-1)}. \tag{9.5}$$

Introducing (9.3) and (9.4) into (9.1) yields the indirect utility function

$$v = Y P^{-\mu} (p^{\mathbb{T}})^{-(1-\mu)}. \tag{9.6}$$

The technology in the $\mathbb{T}$-sector is such that one unit of output requires one unit of L. Each variety of the $\mathbb{M}$-sector is produced according to the same technology such that the production of the quantity $q(i)$ requires $l(i)$ units of skilled labor given by

$$l(i) = f + cq(i), \tag{9.7}$$

where f and c are, respectively, the fixed and marginal labor requirements. Clearly, this technology exhibits scale economies. Without loss of generality, we choose the unit of skilled labor such that $c = 1$. Because preferences exhibit love for diversity and there are increasing returns but no scope economies, each variety is produced by a single firm. Indeed, any firm obtains a higher share of the market by producing a differentiated variety than by replicating an existing one.[9] In turn, this implies that the mass of firms is identical to the mass of varieties and that the output of a firm equals the demand of the corresponding variety.

The output of the $\mathbb{T}$-sector is costlessly traded between any two regions and is chosen as the numéraire, and thus $p^{\mathbb{T}} = 1$. In contrast, the output of the $\mathbb{M}$-sector is shipped at a positive cost according to the "iceberg" technology: when one unit of the differentiated product is moved from region r to region s, only a fraction $1/\Upsilon_{rs}$ arrives at destination, where $\Upsilon_{rs} > 1$ for $r \neq s$ and $\Upsilon_{rr} = 1$. Hence, if variety i is produced in region r and sold at the mill (fob) price $p_r(i)$, the price $p_{rs}(i)$ paid by a consumer located in region s ($\neq r$) is

$$p_{rs}(i) = p_r(i)\Upsilon_{rs}. \tag{9.8}$$

If the distribution of firms is $(M_1, \ldots, M_R)$, using (9.5) and setting $\Upsilon_{rr} = 1$, we obtain the price index P_r in region r from

$$P_r = \left\{\sum_{s=1}^R \Upsilon_{sr}^{-(\sigma-1)} \int_0^{M_s} p_s(i)^{-(\sigma-1)} di\right\}^{-1/(\sigma-1)}. \tag{9.9}$$

Let w_r denote the wage rate of a skilled worker living in region r. Because the price of the traditional good equals 1, the wage of the unskilled workers is also equal to 1 in all regions. Thus, because there is free entry and exit, and therefore zero profit in equilibrium, the income of region r is

$$Y_r = \lambda_r H w_r + \nu_r L. \tag{9.10}$$

From (9.4), the total demand of the firm producing variety i and located in region r is

$$q_r(i) = \sum_{s=1}^R \mu Y_s [p_r(i)\Upsilon_{rs}]^{-\sigma} (P_s)^{\sigma-1} \Upsilon_{rs}$$
$$= \mu p_r(i)^{-\sigma} \sum_{s=1}^R Y_s \Upsilon_{rs}^{-(\sigma-1)} (P_s)^{\sigma-1}. \tag{9.11}$$

This expression requires some comment. The term $\mu Y_s [p_r(i)\Upsilon_{rs}]^{-\sigma} (P_s)^{\sigma-1} \Upsilon_{rs}$ stands for the quantity shipped from the firm located in r to region s. Here, the regional consumption in s, which is equal to $\mu Y_s [p_r(i)\Upsilon_{rs}]^{-\sigma} (P_s)^{\sigma-1}$, must be multiplied by Υ_{rs} because the firm's output "melts" on the way, thus implying that the firm must send out a larger quantity of its output for the desired quantity to be delivered.[10]

Because each firm has a negligible impact on the market, it may accurately neglect the impact of a price change over consumers' income (Y_r) and other firms' prices and hence on the regional price indices (P_r).[11] Consequently, (9.11) implies that, regardless of the spatial distribution of consumers, each firm faces an isoelastic downward-sloping demand (the elasticity equals σ). This very convenient property depends crucially on the assumption of an iceberg transport cost, which affects the level of demand but not its elasticity.

The profit function of a firm in r is

$$\pi_r(i) = p_r(i)q_r(i) - w_r[f + q_r(i)] = [p_r(i) - w_r]q_r(i) - w_r f. \qquad (9.12)$$

Because varieties are equally weighted in the utility function, the equilibrium price is the same across all firms located in region r. Solving the first-order condition using (9.11) yields the common equilibrium price

$$p_r^* = \frac{w_r}{\rho} \qquad r = 1, \dots, R. \qquad (9.13)$$

This means that firms uses a relative markup equal to $1/\rho$, which is independent of the firms' and consumers' distributions. Everything else being equal, more product differentiation leads to a higher markup and, therefore, to a higher equilibrium price. However, the equilibrium price depends on the mass of firms and workers established in region r through the local wage w_r.

Substituting (9.13) into the profit function leads to

$$\pi_r = \frac{w_r}{\sigma - 1}q_r - w_r f = \frac{w_r}{\sigma - 1}[q_r - (\sigma - 1)f]. \qquad (9.14)$$

Under free entry, profits are zero, and thus the equilibrium output of a firm is a constant given by

$$q_r^* = (\sigma - 1)f \qquad r = 1, \dots, R. \qquad (9.15)$$

Note that this quantity is independent of the distributions of firms and workers and is the same across regions. As a result, in equilibrium a firm's labor requirement is also unrelated to the firms' distribution:

$$l^* = \sigma f \qquad r = 1, \dots, R.$$

Thus, the total mass of firms in the M-sector is constant and equal to H/l^*, whereas the corresponding firm distribution

$$M_r = \lambda_r H/l^* = \lambda_r H/\sigma f \qquad r = 1, \dots, R \qquad (9.16)$$

depends only on the distribution of skilled workers. These equalities imply that the core–periphery model allows for the spatial redistribution of the modern sector but not for its growth, for the total number of firms (or varieties) is constant; this issue is addressed in Chapter 11.

Introducing the equilibrium prices (9.13) and substituting (9.16) for M_r in the regional price index (9.9), we obtain

$$P_r = \left[\sum_{s=1}^{R} \frac{\lambda_s H}{\sigma f}\left(\frac{w_s}{\rho}\Upsilon_{sr}\right)^{-(\sigma-1)}\right]^{-1/(\sigma-1)}$$
$$= \kappa_1\left[\sum_{s=1}^{R} \lambda_s (w_s \Upsilon_{sr})^{-(\sigma-1)}\right]^{-1/(\sigma-1)} \qquad r = 1, \dots, R, \qquad (9.17)$$

where

$$\kappa_1 \equiv \rho^{-1}\left(\frac{H}{\sigma f}\right)^{-1/(\sigma-1)},$$

which clearly depends on the spatial distribution of skilled workers as well as on the values of transport costs.

Finally, we consider the labor market clearing conditions for a given distribution of workers. The wage prevailing in region r is the highest wage that firms located there can pay under the nonnegative profit constraint. For that, we evaluate the demand (9.11) as a function of the wage through the equilibrium price (9.13):

$$q_r(w_r) = \mu\left(\frac{1}{\rho}\right)^{-\sigma} w_r^{-\sigma} \sum_{s=1}^{R} Y_s \Upsilon_{rs}^{-(\sigma-1)} P_s^{\sigma-1}. \qquad (9.18)$$

Because this expression is equal to $(\sigma - 1)f$ when profits are zero, we obtain the following implicit expression for the *zero-profit wages*:

$$w_r^* = \kappa_2\left[\sum_{s=1}^{R} Y_s \Upsilon_{rs}^{-(\sigma-1)} P_s^{\sigma-1}\right]^{1/\sigma} \qquad r = 1, \dots, R, \qquad (9.19)$$

where

$$\kappa_2 \equiv \rho[\mu/(\sigma - 1)f]^{1/\sigma}.$$

Clearly, w_r^* is the equilibrium wage prevailing in region r when $\lambda_r > 0$.

Substituting (9.19) for Y and setting $p^{\mathbb{T}} = 1$ in the indirect utility (9.6), we obtain the *real wage* in region r as follows:

$$v_r = \omega_r = \frac{w_r^*}{P_r^{\mu}} \qquad r = 1, \dots, R. \qquad (9.20)$$

Hence, the indirect utility is here equivalent to maximizing the real wage.

Finally, the Walras law implies that the traditional sector market is also in equilibrium provided that the equilibrium conditions above are satisfied.

For a given spatial distribution of skilled workers, we now ask whether there is an incentive for them to migrate and, if so, what direction the flow of migrants will take. A *spatial equilibrium* arises when no skilled worker may get a higher utility level in another region: $(\lambda_1^*, \ldots, \lambda_R^*)$ is a spatial equilibrium if there exists a positive constant ω^* such that

$$\omega_r \leq \omega^* \quad \text{for } r = 1, \ldots, R$$
$$\omega_r = \omega^* \quad \text{if } \lambda_r^* > 0.$$

Hence, the zero-profit real wage that local firms could afford to pay in a region containing no skilled workers is lower than (or just equal to) the equilibrium real wage. Because the functions $\omega_r(\lambda_1, \ldots, \lambda_R)$ are continuous in $(\lambda_1, \ldots, \lambda_R)$ over the compact set

$$\Lambda \equiv \left\{(\lambda_1, \ldots, \lambda_R); \sum_{r=1}^{R} \lambda_r = 1 \quad \text{and} \quad \lambda_r \geq 0\right\},$$

we can appeal to Proposition 1 of Ginsburgh, Papageorgiou, and Thisse (1985) to guarantee that such an equilibrium always exists.

Following a now well-established tradition in migration modeling, we focus on an adjustment process in which workers are attracted (repulsed) by regions providing high (low) utility levels:

$$\dot{\lambda}_r = \lambda_r(\omega_r - \bar{\omega}) \qquad r = 1, \ldots, R,$$

where $\dot{\lambda}_r$ is the time-derivative of λ_r, ω_r is the equilibrium real wage corresponding to the distribution $(\lambda_1, \ldots, \lambda_R)$, and $\bar{\omega} \equiv \sum \lambda_s \omega_s$ is the average real wage across all regions. In other words, the skilled move from the low-wage regions toward the high-wage ones.

A spatial equilibrium is stable if, for any marginal deviation of the population distribution from the equilibrium, the equation of motion above brings the distribution of skilled workers back to the original one. In doing so, we assume that local labor markets adjust instantaneously when some skilled workers move from one region to another. More precisely, the mass of firms in each region must be such that the labor market clearing conditions (9.16) remain valid for the new distribution of workers. Wages are then adjusted in each region for each firm to earn zero profits in any region having skilled workers because workers move toward high-wage regions.

Observe here one more justification for working with a continuum of agents (workers and firms): this modeling strategy allows one to respect the integer nature of a worker's or firm's location while describing the evolution of the regional share of production by means of differential equations.

9.2.2 The Two-Region Case

Consider two regions A and B. The unskilled workers are equally split between regions ($\nu_A = \nu_B = 1/2$). To keep things as symmetric as possible, we also assume that $\Upsilon_{AB} = \Upsilon_{BA} \equiv \Upsilon$. In this specific context, the basic equations developed in the foregoing are as follows:

$$Y_r = \lambda_r H w_r + L/2 \qquad r = A, B \tag{9.21}$$
$$P_r = \kappa_1\left[\lambda_r w_r^{-(\sigma-1)} + \lambda_s(w_s\Upsilon)^{-(\sigma-1)}\right]^{-1/(\sigma-1)} \qquad s \neq r \tag{9.22}$$
$$w_r^* = \kappa_2\left(Y_r P_r^{\sigma-1} + Y_s \Upsilon^{-(\sigma-1)} P_s^{\sigma-1}\right)^{1/\sigma} \qquad s \neq r \tag{9.23}$$
$$\omega_r = w_r^* P_r^{-\mu} \qquad r = A, B. \tag{9.24}$$

Whenever this turns out to be convenient, from now we use $\lambda \equiv \lambda_A$ so that $\lambda_B = 1 - \lambda$. Given a parametric solution to the system (9.21)–(9.24), a *spatial equilibrium* arises at $\lambda \in (0, 1)$ when

$$\Delta\omega(\lambda) \equiv \omega_A(\lambda) - \omega_B(\lambda) = 0$$

or at $\lambda = 0$ when $\Delta\omega(0) \leq 0$, or at $\lambda = 1$ when $\Delta\omega(1) \geq 0$.

The stability is studied with respect to the following equation of motion:

$$\dot{\lambda} = \lambda\Delta\omega(\lambda)(1 - \lambda) \tag{9.25}$$

and is defined as in Section 8.4.[12] If $\Delta\omega(\lambda)$ is positive and $\lambda \in (0, 1)$, workers move from B to A; if it is negative, they go in the opposite direction. Clearly, any spatial equilibrium is a steady-state for (9.25).

The system (9.21)–(9.24) of nonlinear equations cannot be solved analytically. As a consequence, deriving a characterization of its solution in terms of λ is not simple. To derive some insight into the nature of the equilibrium, computational experiments have been performed by Krugman (1991a).[13] The results are displayed in Figure 9.1, where the following results appear. For a large value of Υ ($= \Upsilon_1$), there is only one equilibrium corresponding to the full dispersion of the modern sector ($\lambda = 1/2$), which is stable. When Υ takes some intermediate value Υ_2, four more equilibria emerge that are all asymmetric. However, the two interior equilibria are unstable. Hence, three stable equilibria now exist: the symmetric configuration and the core–periphery structure with concentration of the modern sector in region A or region B. Finally, when Υ takes a sufficiently low value ($= \Upsilon_3$), the symmetric equilibrium becomes unstable, and thus the core–periphery structure is the only stable outcome.

These observations will serve as a guide in the rest of the analysis.

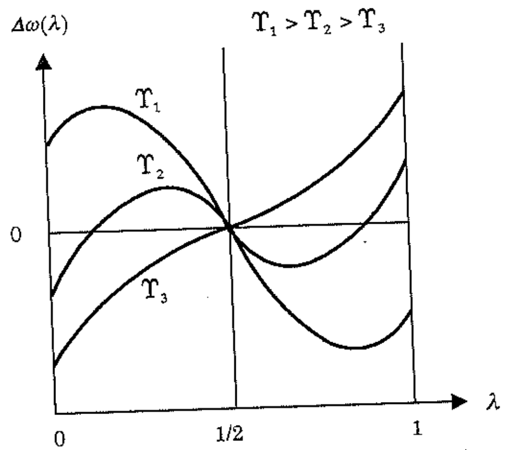

Figure 9.1: Migration dynamics under various values of Υ.

9.2.3 The Core–Periphery Structure

Suppose that the modern sector is concentrated in one region, say region A so that $\lambda = 1$. To check whether this is an equilibrium, we ask whether a skilled worker could be strictly better off in B. More precisely, we wish to determine conditions under which the real wage he may obtain in region B does not exceed the real wage this worker gets in region A. Setting $\lambda = 1$ in (9.21)–(9.24), we get the following equations:

$$Y_A = Hw_A^* + L/2 \quad \text{and} \quad Y_B = L/2$$
$$P_A = \kappa_1 w_A^* \quad \text{and} \quad P_B = \kappa_1 \Upsilon w_A^*. \tag{9.26}$$

Then, w_A^* is obtained by substituting (9.16) into (9.23) with $r = A$,

$$w_A^* = \kappa_2 \left[Y_A (\kappa_1 w_A^*)^{\sigma-1} + Y_B \Upsilon^{-(\sigma-1)} (\kappa_1 \Upsilon w_A^*)^{\sigma-1} \right]^{1/\sigma}$$

which yields $w_A^* = (\mu/H)(Y_A + Y_B)$, or

$$w_A^* = \frac{\mu}{1-\mu} \frac{L}{H}.$$

From (9.13), it is then possible to determine the common equilibrium price of all varieties in terms of the fundamentals of the economy:

$$p_A^* = \frac{1}{\rho} \frac{\mu}{1-\mu} \frac{L}{H},$$

which shows that the price of the differentiated product within the agglomeration increases with the unskilled–skilled ratio (L/H) as well as with the share of the modern sector (μ).

Finally, when the modern sector is geographically concentrated in A, the regional nominal incomes are as follows:

$$Y_A = \frac{\mu}{1-\mu} L + \frac{L}{2} \quad \text{and} \quad Y_B = \frac{L}{2},$$

and thus the gross domestic product (GDP) of the economy is given by

$$Y_G \equiv Y_A + Y_B = L/(1-\mu).$$

The equilibrium real wage in region A is

$$\omega_A = \kappa_1^{-\mu} (w_A^*)^{1-\mu} = \left(\frac{1}{\rho}\right)^{-\mu} \left(\frac{H}{\sigma f}\right)^{\mu/(\sigma-1)} \left(\frac{\mu}{1-\mu}\frac{L}{H}\right)^{1-\mu},$$

which is independent of Υ.

Agglomeration in region A is an equilibrium if and only if ω_A is larger than or equal to ω_B. Thus, we need to determine ω_B. To find it, we substitute (9.22) for the price index and (9.23) for the nominal wage into the real wage (9.24) and get

$$\omega_B = \kappa_1^{\rho-\mu} \kappa_2 (w_A^*)^{\rho-\mu} \Upsilon^{-\mu} \left(Y_A \Upsilon^{-(\sigma-1)} + Y_B \Upsilon^{\sigma-1}\right)^{1/\sigma}$$
$$= \kappa_1^{\rho-\mu} \kappa_2 (w_A^*)^{\rho-\mu} \Upsilon^{-\mu} \left(Y_A \Upsilon^{-(\sigma-1)} + Y_B \Upsilon^{\sigma-1}\right)^{1/\sigma}.$$

It can then readily be verified that

$$\frac{\omega_B}{\omega_A} = \left[\frac{1+\mu}{2} \Upsilon^{-\sigma(\mu+\rho)} + \frac{1-\mu}{2} \Upsilon^{-\sigma(\mu-\rho)} \right]^{1/\sigma}. \tag{9.27}$$

When shipping is costless ($\Upsilon = 1$), we always have $\omega_B/\omega_A = 1$: location does not matter. Furthermore, the first term in the right-hand side of (9.27) is always decreasing in Υ. Therefore, because the second term is also decreasing when $\mu \geq \rho$, the ratio ω_B/ω_A always decreases with Υ, thus implying that $\omega_B < \omega_A$ for all $\Upsilon > 1$. This means that the core–periphery structure is a stable equilibrium for all $\Upsilon > 1$. When

$$\mu \geq \rho,$$

which is called the *black hole condition*, varieties are so differentiated that firms' demands are not very sensitive to differences in transportation costs, thus making the agglomeration force very strong. In fact, it is so strong that agglomeration can be viewed as a "black hole" attracting any movable activity.

More interesting is the case in which

$$\mu < \rho, \tag{9.28}$$

that is, varieties are not very differentiated so that the firms' demands are sufficiently elastic and, hence, the agglomeration force is weaker. If (9.28)

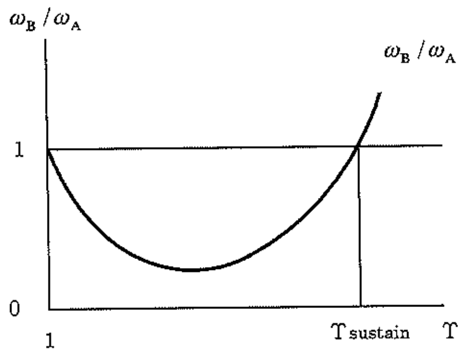

Figure 9.2: The determination of the sustain point.

holds, then the second term in (9.27) goes to infinity when $\Upsilon \to \infty$, and the ratio ω_B/ω_A is as depicted in Figure 9.2.[14]

We see that a single value $\Upsilon_{\text{sustain}} > 1$ exists such that $\omega_B/\omega_A = 1$. Hence, the agglomeration is a stable equilibrium for any $\Upsilon \leq \Upsilon_{\text{sustain}}$. In other words, once all firms belonging to the modern sector locate together within a region, they stay there as long as carrying their output to the other region is sufficiently cheap.[15] This occurs because firms can enjoy all the benefits of agglomeration without losing much of their business in the other region. Such a point is called the *sustain point* because, once firms are fully agglomerated, they stay so for all smaller values of Υ.[16] On the other hand, when transportation costs are sufficiently high ($\Upsilon > \Upsilon_{\text{sustain}}$), firms lose much on their exports so that the core–periphery structure is no longer an equilibrium.

Summarizing these results, we have the following:

Proposition 9.1 *Consider a two-region economy.*

1. *If $\mu \geq \rho$, then the core–periphery structure is always a stable equilibrium.*
2. *If $\mu < \rho$, then a unique solution $\Upsilon_{\text{sustain}} > 1$ exists to the equation*

$$\frac{1+\mu}{2}\Upsilon^{-\sigma(\mu+\rho)} + \frac{1-\mu}{2}\Upsilon^{-\sigma(\mu-\rho)} = 1 \tag{9.29}$$

such that the core–periphery structure is a stable equilibrium for any $\Upsilon \leq \Upsilon_{\text{sustain}}$.

It is remarkable that $\Upsilon_{\text{sustain}}$ depends only on the degree of product differentiation (σ) and the share of the modern sector in consumption (μ).

Interestingly, Proposition 9.1 provides formal support of the claim made by Kaldor (1970, 241) more than 30 years ago:

When trade is opened up between them, the region with the more developed industry will be able to supply the need of the agricultural area of the other region on more favourable terms: with the result that the industrial centre of the second region will lose its market and will tend to be eliminated.

The proposition also supports the claim of Giersch (1949, 94), who observed more than half a century ago that

production would tend to be centered in those industrial countries which already provide large domestic markets before the formation of the federal state.

It is worth stressing that the agglomeration is obtained as the aggregate outcome of a handful of individual decisions: the skilled workers do not choose a priori to be (or not to be) together. They are brought together through individual decisions based on current market prices and wages.

9.2.4 The Symmetric Structure

What we have just seen suggests that the modern sector is geographically dispersed when transportation costs are high and when (9.28) holds. To check this conjecture, we consider the symmetric configuration ($\lambda = 1/2$). In this case, there are only four equilibrium conditions,

$$Y_A = Y_B = Y = (H/2)w^* + L/2,$$

where w^* is the common zero-profit wage prevailing at the symmetric configuration given by

$$w^* = \kappa_2\left(YP^{\sigma-1} + Y\Upsilon^{-(\sigma-1)}P^{\sigma-1}\right)^{1/\sigma}$$
$$= \kappa_2(YP^{\sigma-1})^{1/\sigma}\left(1+\Upsilon^{-(\sigma-1)}\right)^{1/\sigma},$$

and the common price index is equal to

$$P = \kappa_1\left[\frac{1}{2}(w^*)^{-(\sigma-1)} + \frac{1}{2}(w^*\Upsilon)^{-(\sigma-1)}\right]^{-1/(\sigma-1)}$$
$$= \kappa_1 2^{1/(\sigma-1)}w^*\left(1+\Upsilon^{-(\sigma-1)}\right)^{-1/(\sigma-1)}.$$

The common real wage is

$$\omega = w^* P^{-\mu}.$$

Because $\omega_A = \omega_B = \omega$, the symmetric structure is a spatial equilibrium for all $\Upsilon > 1$.

For a given $\Upsilon > 1$, the symmetric equilibrium is stable (unstable) if the slope of $\Delta\omega(\lambda)$ is negative (positive) at $\lambda = 1/2$. Checking this condition requires fairly long calculations using all the equilibrium conditions. However, Fujita, Krugman, and Venables (1999, chap. 5) have shown the following results. First, when (9.28) does not hold, the symmetric equilibrium is always unstable. However, when (9.28) holds, this equilibrium is stable (unstable) if Υ