Knowledge Spillovers in Cities: An Auction Approach

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Abstract: This paper proposes a micro-foundation for knowledge spillovers. I model a city in which free knowledge transfers are bids by experts to entrepreneurs who auction jobs. These knowledge bids resemble a consultant's pitch to a potential client. Two fundamental properties of knowledge underlie the model: First, it is often necessary to reveal some knowledge to demonstrate its value. Second, knowledge is freely reproducible. Larger cities generate more meetings between experts and entrepreneurs, resulting in more learning and better matches. Larger cities also foster competition for jobs, which motivates experts to raise their knowledge bids. These results demonstrate how competitive behavior can be a source of agglomeration economies, and contribute to explain the higher productivity of urban workers.

Key words: agglomeration, auction, knowledge spillovers.

JEL classification: D44, D83, R23, R39

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1. Introduction

Urban environments have the potential to catalyze learning and networking activity. In Alfred Marshall's influential argument, chance encounters and imitation take place whenever individuals live in close proximity, and knowledge spillovers in cities happen quite inadvertently.[1] Intuitively, however, knowledge spillovers also result from deliberate individual decisions (Helsley and Strange, 2004). The motivation behind these individual decisions remains little understood,[2] in the face of growing evidence on the crucial role that knowledge spillovers play in shaping the geography of production in modern economies.

The main objective of the paper is to explain why people sometimes let their knowledge 'spill', instead of selling it in markets. I model uncompensated knowledge transfers (knowledge spillovers) as bids by experts to entrepreneurs who auction jobs. An expert could be a consultant pitching an idea to a potential client, such as free advice on how to advertise a product. Unlike extant literature that focuses on imitation and reciprocity, this explanation for knowledge spillovers relies on competitive behavior and on the fundamental properties of knowledge as an input. I use such knowledge spillovers as a micro-foundation for agglomeration economies, to show how large cities improve knowledge diffusion and offer better job matches between experts and entrepreneurs. Interestingly, heightened competition for jobs enhances experts' willingness to transfer free knowledge, providing a new explanation for the higher productivity of workers in large and dense urban areas.[3]

Interest in urban knowledge diffusion stems from the presumption that density facilitates face-to-face transfers of productive ideas. Patent citations provide direct evidence that physical proximity favors knowledge diffusion, and Jaffe, Trajtenberg, and Henderson (1993) find that a patent holder is more likely to cite other patents from inventors who are geographically close. The importance of face-to-face interactions is revealed in the very localized nature of production

[1] See Marshall (1920). Glaeser (1999) provides a model formalizing this idea, in which knowledge flows through imitation of the old by the young, with the old getting a share from the young's returns to a successful skill transfer.

[2] Puga (2010), in his review of the literature, argues that: "We have good models of agglomeration through sharing and matching, but not a deep enough understanding of learning in cities."

[3] A vast empirical literature in urban economics establishes that firms and individuals in larger agglomerations are more productive; see Puga (2010) for a review of the literature. The classic explanations for this productivity advantage are so-called 'agglomeration economies'; processes through which interactions between firms or individuals are facilitated in cities, in a way that makes these units more productive.

externalities.[4] Rosenthal and Strange (2008) and Arzaghi and Henderson (2008) find that the productivity gains from spatial proximity to other people decay sharply with distance. This decay suggests that face-to-face interactions, whose cost is especially sensitive to distance, are prominent among the factors making people in larger and denser cities more productive.

Wage regressions corroborate the importance of learning in cities. Workers in large cities earn on average 30% more than workers in rural areas (Glaeser, 2011), but this premium does not immediately accrue to a worker upon moving to a bigger city. Much of these earning gains arise only as a worker accumulates experience in the city (Glaeser and Maré, 2001, De la Roca and Puga, 2013). Recent estimates from Spain show that about half of the earning advantage of urban workers is acquired through time (De la Roca and Puga, 2013), which suggests that learning plays a central role in increasing urban workers' productivity. Jacobs (1968), Lucas (1988), and others take the argument further and conclude that in a knowledge-based economy, idea transfers in cities are fundamental to economic growth.

I develop a model in which individuals meet in a city, and then start to work. During a meeting, an expert chooses how much knowledge to freely transfer to an entrepreneur. After learning from all experts, an entrepreneur hires one of them and produces a good with a constant returns to scale technology, using both the knowledge obtained during meetings and that acquired from the hired expert. The model's intellectual foundation and defining assumptions – that the value of knowledge is unobservable and communicated at no cost – derive from two key ideas about the properties of knowledge as an input. The first idea, from Arrow (1962), is that one often must possess knowledge to assess its value, or reveal an idea to demonstrate its usefulness. This is unlike a physical object like a car, which can be taken for a test drive and brought back. The second idea is that knowledge is freely reproducible, or has a negligible marginal cost of production once a blue-print exists.[5]

Arrow's property suggests the motivation behind uncompensated knowledge transfers. For an expert, a free transfer is a bid for a job contract, a means to prove how knowledgeable she is. For an entrepreneur, who auctions a job to experts, the free knowledge transfers are an opportunity to

[4]Few studies are able to directly assess the importance of knowledge spillovers relative to other agglomeration forces. Rosenthal and Strange (2001) find evidence supporting all three sources of agglomeration identified by Marshall (1920) - input sharing, labor pooling and knowledge spillovers - with labor pooling being the best predictor of industry agglomeration. Ellison, Glaeser, and Kerr (2010) also find evidence for all three sources by looking at coagglemeration patterns for different industries, but find that input sharing (specifically, proximity to suppliers and customers) having the most explanatory power.

[5]The first model relating increasing returns to the free reproducibility of knowledge is from Romer (1990).

accumulate knowledge, and to find the best expert for a particular job. An auction framework delineates the strategic incentives to give some valuable knowledge without direct payment, but not all. It identifies conditions under which knowledge 'spills' in non-market interactions, as opposed to market transactions occurring after the best expert for a job is revealed and starts working for an entrepreneur. Agglomeration economies result from free reproducibility of knowledge and the greater number of meetings between experts and entrepreneurs in a larger city. In particular, the model uncovers a new agglomeration force, stemming from competitive behavior: As the number of experts competing for jobs in a city increases, their willingness to freely transfer knowledge – the size of their knowledge bid – also increases. Dividing agglomeration forces into sharing, matching and learning as in Duranton and Puga (2004), larger cities in this model stimulate more learning and better matches.

Auction theory offers intuitive models of people's behavior in competitive environments, which reward individuals for being smarter and more knowledgeable than others.[6] As such, the model evokes so-called 'industrial clusters ,' in which networking is essential, job changes are frequent and, as shown in Freedman (2008), workers initially accept lower wages while they develop their reputations. The model also captures the networking behavior of workers in finance, advertising, law, graphic design and other consulting industries, as they 'pitch' their ideas to potentials clients.[7]

The paper contributes to several strands of literature in urban economics, innovation, knowledge markets and mechanism design (auctions). The innovation literature, unlike this paper, emphasizes the role of reciprocity in fostering incentives to engage in free knowledge transfers. In a seminal paper, von Hippel (1987) uses survey evidence to document how the expectation of reciprocal transfers motivates engineers in the steel industry to share knowledge with their peers. With the exception of Glaeser (1999), who focuses on imitation, reciprocity is also the motivation for knowledge transfers in theoretical models of knowledge diffusion and creation. Jovanovic and Rob (1989) propose a model in which agents can develop ideas privately, but can also share their ideas with other agents, such interactions leading both to imitation (diffusion) and invention (creation). Berliant and co-authors (2006, 2008 and 2009) refine Jovanovic and Rob's insight that

[6] At another extreme, Niehaus (2011) assumes that people transfer knowledge non-strategically, as long as the benefits to the learner exceed the costs to the sender.

[7] While Marshall (1920) emphasizes within-industry externalities, Jacobs (1968) highlights the importance of cross-industry spillovers. Glaeser, Kallal, Scheinkman, and Shleifer's (1992) finding that a diverse industry mix is more conducive to industry growth supports the preeminence of Jacobian externalities. Henderson, Kunkoro, and Turner (1995), however, find that Marshallian externalities are more important, at least for mature industries.

new and better ideas originate from the contact of different ideas, and provide an analysis of the costs and benefits of urban diversity for knowledge creation. Helsley and Strange's (2004) model is the first in which the choice of how much knowledge to transfer is endogenous. In their model, agents living in a city can barter knowledge and develop a reputation for cooperation, in line with von Hippel's evidence on 'know-how' sharing. Finally, Davis and Dingel (2012) consider idea exchange as an agglomeration force in a system of cities model.

The expectation of reciprocity certainly motivates many free knowledge transfers, but my model purposely excludes the possibility of sharing or reciprocity. I provide an alternative explanation for uncompensated knowledge transfers, based on the incentives to communicate how knowledgeable one is. Supporting this explanation, Appleyard (1996) finds that reciprocity is less important in high-tech or other rapidly evolving industries, precisely the kind of industries in which agglomeration economies are strongest (Moretti (2004), Fu (2007), and others). Ignoring reciprocity reconciles theories on knowledge spillovers with these empirical findings.

The paper also relates to the literature on knowledge markets. Bhattacharya and Ritter (1983), Anton and Yao (2002), and others draw from Spence's (1973, 1974) work on market signaling to show that Arrow's property can lead firms to provide a 'voluntary disclosure of knowledge' (another expression for 'free knowledge transfers'). My model differs from existing models of voluntary knowledge disclosure, in that it defines a free knowledge transfer not as a 'signal', but rather as a 'bid', a simpler concept that is better suited to informal and semi-formal interactions in competitive urban environments.

Finally, the paper relates to the literature on auction theory. The solution to the basic version of the model only involves the simplest case of a first-price auction with no entry cost. However, knowledge auctions in an urban environment differ in two ways from existing auction models. First, a knowledge auction is all-pay from the auctioneer's perspective, but not from that of the bidder. This implies, for instance, that unlike in a standard first-price auction, a positive reservation price (i.e. requiring a minimum knowledge bid) is not optimal for an entrepreneur. Second, entry costs in urban environments are endogenous, because congestion leads to higher meeting costs in larger cities.

The remainder of the paper is organized as follows: Section 2 presents and interprets the knowledge auction in a closed city, along with industry examples that corroborate the model. Section 3 analyzes the equilibrium size and composition of an open city for the case in which experts, who differ in the value of their knowledge, choose to migrate to a city *before* learning their type. Section 4 also analyzes an open city, but in which experts choose to migrate *after* learning their type, which leads to sorting of the best experts into the city.[8] In this case, an increase in the cost of urban living generates lower knowledge bids, and a trade-off between the quality and the quantity of experts moving to the city. This last model is a generalization with endogenous entry cost of a simpler model in which each meeting carries a fixed communication cost. Section 5 concludes.

2. A model of knowledge transfers

Consider a closed city with N inhabitants, a number X of which are experts who hold some knowledge, and a number E of which are entrepreneurs who can use that knowledge in production, so that $X + E = N$.

Experts and entrepreneurs play a two-stage game: first a meeting stage, then a production stage. At the meeting stage, experts learn their type and choose how much knowledge to freely transfer to entrepreneurs that they meet in the city. Entrepreneurs learn from the knowledge transferred by experts. At the production stage, each entrepreneur hires one expert and pays her for the knowledge that she has not already given at the meeting stage.[9] Entrepreneurs then produce a consumption good, using both the knowledge accumulated at the meeting stage and that acquired at the production stage of the game.

Let i index the set of X experts, and j index the set of E entrepreneurs. The value of expert i's knowledge to entrepreneur j is denoted by k_{ij}, so expert i's type is a E-vector of knowledge $k_i = (k_{i1}...,k_{ij},...k_{iE})$. k_{ij} is independently and identically distributed over the [0,1] uniform distribution, for all $i \in \{1,...,X\}$ and $j \in \{1,...,E\}$. The value of an expert's knowledge to an entrepreneur is independent of its value to another entrepreneur, to reflect the idea that different experts are proficient in different fields, and that each entrepreneur puts a distinct value on any expert's knowledge.[10] Knowledge has two properties: there is no cost of communicating it (free

[8] Combes, Duranton, and Gobillon (2008) find strong evidence that more productive workers sort into larger cities, and recent empirical work by De la Roca and Puga (2013) attribute most of these urban workers' higher productivity to their ability to learn faster after moving to a city.

[9] Appendix B solves a version of the model in which an entrepreneur can hire many experts, and provides numerical evidence that an entrepreneur prefers to hire just one.

[10] This independence does not affect any of the results in this section. It only matters in Section 4, in which experts can choose to migrate into an open city after learning their type.

reproducibility), and the value of an expert's knowledge is unobservable to entrepreneurs or other experts. The meeting technology is such that every expert meets every entrepreneur in the first stage of the game, so the number of meetings is $E \times X$. There is no cost of meeting, an assumption that I relax in Section 4. Define $b_{ij}(k_{ij}, X)$ as the knowledge transferred for free by expert i to an entrepreneur j that she meets in the first stage of the game. The notation anticipates that this transfer will be function of X and k_{ij}. Each entrepreneur uses the same constant returns to scale, additive production function to produce a single good y, with knowledge as the only input. Entrepreneur j produces an amount y_j of the good, which is equal to the knowledge received for free at the meeting stage of the game, plus the knowledge paid for at the production stage (denoted by x_j), so that:

$$y_j = \left(\sum_{i=1}^{X} b_{ij}(k_{ij}, X) \right) + x_j. \tag{1}$$

The utility function of both experts and entrepreneurs is $u(y) = y$, where y is the amount of the good consumed.

The wage setting mechanism is such that an expert earns a wage equal to his productivity at work, x_j, paid for in units of the good.[11] This wage contract depends on the actual surplus created by an expert, not on the belief of an entrepreneur about her type. Given the CRS, additive production function, it follows that expert i hired by entrepreneur j earns a wage equal to $k_i - b_{ij}(k_{ij}, X)$; the value of the knowledge that she has not already given during the meeting stage of the game. Knowledge transferred for free cannot be given back and as such is never paid for. An expert can work for any number of entrepreneurs, and her wage is the same as her utility from obtaining a job, because of the linear utility function.

It only remains to specify the job allocation mechanism. For an entrepreneur, the unobservability of experts' knowledge presents an opportunity to ask for free knowledge, with a job offered as a reward. A particularly attractive mechanism is the simple rule awarding a job to the expert who transfers the most knowledge during the meeting stage. As the next section demonstrates, this rule (which turns out to be a first price-auction) allows an entrepreneur to find the best expert for a particular job, as well as to obtain free knowledge from all other experts.[12] This first-price auction is arbitrarily close to any optimal mechanism as the number of experts becomes large, in the sense

[11] Equivalently, the wage is settled through Nash bargaining over the surplus produced by a working expert, and λ, the bargaining power of an entrepreneur, is equal to zero. A different value of λ would not affect any of the main results of this section.

[12] Entrepreneurs do not need to commit to hiring only one expert in a Bayesian equilibrium, because they do not get a share λ of experts' surplus and so ex-post they are indifferent between hiring zero, one or many experts.

that an entrepreneur freely uses the knowledge of all experts. In the appendix, I provide evidence that this job allocation mechanism is optimal under two important extensions of an entrepreneur's action space. First, Appendix B allows entrepreneurs to require a minimum knowledge transfer, and shows that they prefer to set it to zero. This result distinguishes knowledge auctions from standard auctions in which an auctioneer can increase its expected payoff by setting a positive reservation price (Myerson, 1981). Second, Appendix A allows entrepreneurs to hire any number of the top experts - who transferred the most knowledge during meetings - and finds that they prefer to hire only one expert (this last result, however, is only supported by numerical evidence). Of course, there exist plausible extensions of an entrepreneur's action space under which the first-price auction is not optimal, for instance if entrepreneurs can make additional transfer payments beyond the wage setting mechanism.[13] The first-price auction, however, is probably a more realistic representation of entrepreneurs' behavior in informal or semi-formal urban meetings than the richer incentive schemes suggested by optimal mechanism design with larger action spaces.

2.1 *Interpretation: auction framework*

Auction theory is relevant to knowledge transfers in cities because its main tenets have a plausible interpretation within the context of urban working and networking environments. First, an auction implies that a free transfer is a bid, rather than a signal, a reciprocal transfer, or something else. In competitive urban environments, individuals care about recognition as more knowledgeable than their rivals for jobs, and a bid will often capture the motivation behind knowledge transfers better than a purely reciprocal exchange. This may explain why studies find reciprocity to be less important in industries featuring the strongest agglomeration economies. While extensions of Spence (1973) signaling model can explain voluntary knowledge disclosure for firms (e.g. Bhattacharya and Ritter 1983), the relative complexity of signaling games lessens their relevance to informal and semi-formal interactions, and makes them less amenable to modeling competitive environments.[14]

[13] Appendix C shows that entrepreneurs can increase their expected utility by offering a wage top-up to an expert winning the auction.

[14] In a basic auction model, individuals with higher valuation for a good (in this paper, a job) bid more. Signaling games, however, generally feature multiple equilibria, as the cost of sending a given message may be lower for high types, but the informativeness of this message depends on whether low types imitate it or not. Moreover, auction theory's straightforward predictions on the relationship between the number of bidders and the size of their bids, which are important to this paper on agglomeration, have no parallel in the signaling literature, which almost always feature one sender. One recent exception is from Barraquer and Tan (2012), who study competition in a signaling model with multiple senders. They show that when the number of senders is large enough, the only equilibrium satisfying a selection criterion is that in which all agents send the most informative message.

Second, an auction implies that the value of an expert's knowledge corresponds to his valuation for a job. This condition holds if more knowledgeable experts, who perform better at work, earn larger rewards. Most industries feature some degree of performance pay, more so in agglomerated sectors like technology, professional services (finance, law, marketing, etc) and consulting. I argue later that such sectors provide the clearest real-world counterpart to my model.

Third, an auction implies that the number of experts competing for a job affects the size of their knowledge bid. Intuitively, more intense competition for jobs should improve people's disposition towards free knowledge transfer. This prediction, however, really depends on the notion that one cannot be paid for knowledge already transfered for free, i.e. that knowledge is expropriable. In this case, there are clear incentives to transfer just enough knowledge to outbid all other experts, and the number of competing experts matters. In theory, expropriability is just a corollary of Arrow's property (Anton and Yao, 2002). Empirically, expropriability is hard to verify. In some industries, ethical concerns, social norms or regulations supercede entrepreneur's pecuniary interest in expropriating experts.[15] I provide industry examples after solving the model.

2.2 *Results*

The previous section defined the job allocation mechanism (an auction) and the production stage of the game, so it is now possible to work backwards and solve for an expert's equilibrium strategy at the meeting stage of the game. Using auction terminology, k_{ij} becomes expert i's valuation for a job with entrepreneur j, and $b_{ij}(k_{ij},X)$ becomes her bid for a job. An expert optimally works for every entrepreneur who offers her a job, and the E auctions that she participates in are strategically independent (so the subscript j on the bidding function becomes superfluous). The strategy of an expected utility maximizing expert i, when meeting entrepreneur j, is to transfer the amount of free knowledge that maximizes the probability that she gets the job times the utility (wage) that she obtains if she is hired. If all other experts use the same bidding function b, i.e. in a symmetric equilibrium, then the problem of an expert of type $k_{ij} \in [0,1]$ is to submit a bid $b_i \geq 0$ that solves:

$$\max_{b_i} prob \left(b_i > \max_{l \in \{1,\ldots,X\} \setminus \{i\}} b(k_{lj},X) \right) \times (k_{ij} - b_i). \tag{2}$$

[15] There is certainly much anecdotal evidence, for instance, that designers providing free designs in the hope of winning a contest are paid below market rates. For a discussion see the article by Michelle Goodman *When to work for nothing* in the New York Times 'Shifting Careers' blog, November 9, 2008 (http://shiftingcareers.blogs.nytimes.com/2008/11/09/when-to-work-for-free/, retrieved 10 August 2013).

Equation (2) demonstrates that for an expert, a meeting is equivalent to participation in a first-price auction. The highest bidder wins and 'pays' his bid, because an expert cannot be paid at the production stage for the knowledge already transfered at the meeting stage. The solution to an expert problem is well-known, as $b(k_{ij},X) = ((X-1)/X)k_{ij}$ is the unique symmetric Nash equilibrium of a first-price auction in the independent private value model with quasi-linear utility and a $U[0,1]$ distribution of valuation, see for instance Jehle and Reny (2000).[16] Lemma 1 formalizes these results.

Lemma 1 *For an expert, a meeting is equivalent to participation in a first-price auction that requires solving equation* (2)*. There exists a unique symmetric Bayesian Nash equilibrium bidding function that is strictly increasing in k and with* $b(0,X) = 0$*, given by* $b(k,X) = \frac{X-1}{X}k$.

Proof Appendix D derives the bidding function in a more general case with entry costs (see the proof of Lemma 3). □

An important corollary of Lemma 1 is that each entrepreneur k hires the expert that is best for him, i.e. with the highest knowledge value k_{ij} from the set $\{k_{1j},\ldots,k_{Xj}\}$. This is a consequence of a bidding function that strictly increases in k, for all $X > 0$. Another key property of the bidding function is that $\frac{\partial b}{\partial X} > 0$, for all $k \in [0,1]$ and all $X > 0$ (I remove subscripts when no confusion if possible). That is, the value of the free knowledge transferred by each type of experts increases with the number of other experts competing for jobs in the city. Before stating and proving the main proposition of this section on total production in a closed city, I derive a number of intermediate results.

To compute $\mathbb{E}(x)$, the expected wage of an expert, define a random variable $k_{(X)}$ representing the highest order statistic of X draws from the *i.i.d.* $U[0,1]$ distribution, with probability distribution function $f(k_{(X)}) = Xk_{(X)}^{X-1}$. The expected wage is then:

$$\begin{aligned} \mathbb{E}(x) &= \int_0^1 \left(k_{(X)} - b(k_{(X)},X)\right) f(k_{(X)})dk_{(X)} \\ &= \int_0^1 \left(k_{(X)} - \frac{X-1}{X}k_{(X)}\right) Xk_{(X)}^{X-1}dk_{(X)} \\ &= \frac{1}{X+1}, \end{aligned} \tag{3}$$

[16]The result holds for all quasi-linear utility functions, so considering Nash bargaining for wages with λ as the bargaining power of an entrepreneur would not affect the bidding function of experts, which stays the same for wages equal to $(1-\lambda) \times (k - b(k))$ and $0 \leq \lambda < 1$.

where the first line is simply the expected value of the best expert's knowledge (highest order statistics) minus her bid, and the second equality substitutes for the bidding function.

The expected utility of an expert can be computed both before and after she learns her type (but note that an expert always knows her type when she bids). Such distinction is useful in extensions of the models in which experts can migrate to open city, covered in Section 3 and 4. The expected utility of an expert i after learning her type k_i is equal to:

$$\begin{aligned}&\mathbb{E}\left(\sum_{j=1}^{E}\left(k_{ij}-b(k_{ij},X)\right)\mathbb{I}\left[k_{ij}>k_{hj},\forall h\in\{1,...,X\}\backslash\{i\}\right]\right) \\ &=\sum_{j=1}^{E}k_{ij}^{X-1}\left(k_{ij}-b(k_{ij},X)\right) \\ &=\sum_{j=1}^{E}\frac{k_{ij}^{X}}{X},\end{aligned} \tag{4}$$

where $\mathbb{I}\{\cdot\}$ is an indicator function equals to 1 if expert i wins a job from entrepreneur j. The first equality obtain because $\mathbb{E}$ is a linear operator, and k_{ij}^{X-1} is the probability that expert i wins the auction of entrepreneur j. The second equality comes from substituting the bidding function. Equation (4) shows that after learning her type, the expected utility of an expert increases with the number of entrepreneurs and with the value of her knowledge, and decreases with the number of competing experts.

The expected utility of an expert before learning her type is equal to:

$$\frac{1}{1+X}\times\frac{E}{X}, \tag{5}$$

the expected wage from equation (3), times E/X, the number of jobs that each expert can expect to win. Therefore, the expected utility of an expert before she learns her type increases with the number of entrepreneurs and decreases with the number of other experts.

To find the expected utility of an entrepreneur, remind that they do not get a share of the surplus produced by working experts. So an entrepreneur j's expected utility is equal to the expected value of the knowledge transferred during meetings:

$$\mathbb{E}\left(\sum_{i=1}^{X}b(k_{ij})\right)=X\int_0^1 b(z)dz=X\int_0^1\frac{X-1}{X}zdz=\frac{X-1}{2}. \tag{6}$$

This function displays increasing returns to the number of experts. Increasing returns arise because of the competitive behavior of experts, who bid more as the number of other experts increases, and because of the special nature of knowledge. That is, the knowledge auction is *all-pay* from the perspective of an entrepreneur, who also makes productive use of the knowledge bid of experts that he does not hire. The expected utility of an entrepreneur does not depend on E, as there is no competition between entrepreneurs in the model.

Finally, an expression for expected total production in the city results from simple arithmetics.

Proposition 1 *Expected total production in a closed city is equal to* $Y = E\left(\frac{X-1}{2} + \frac{1}{X+1}\right)$

Proof The term inside the bracket is expected production per entrepreneur, which is equal to the expected sum of the knowledge transfered to each entrepreneur in the meeting stage of the game (equal to $(X-1)/2$, from equation 6), plus the expected knowledge used by each entrepreneur during the production stage (equal to $1/(X+1)$, the expected wage paid by an entrepreneur, from equation 3). All entrepreneurs are identical, so expected total production is equal to E times per entrepreneur production. □

The defining feature of total expected production is the presence of external returns to scale. That is, doubling the number of experts and entrepreneurs in the city more than doubles Y, despite the production function itself having constant returns to scale. There are two sources of increasing returns to the number of city inhabitants: an increase in the number of meetings (extensive margin) and an increase in the the productivity of each meeting (intensive margin).

First, increasing returns at the extensive margin come from the matching function, which displays increasing returns to scale, because it matches each entrepreneur with each expert. Second, increasing returns at the intensive margin originate from experts' choice to reveal more knowledge during meetings as competition for jobs heats up. This is a standard auction result; bids increase with the number of auction participants. Note that at the intensive margin, there are only increasing returns to X, the number of experts in the city.[17] These returns vanish as X becomes large and experts bid the entire value of their knowledge at the meeting stage ($\lim_{X\to\infty} b(k) = \lim_{X\to\infty}((X-1)/X)k = k$). In this case, the auction mechanism is necessarily optimal for entrepreneurs, who extract all the knowledge of every expert during meetings. The

[17]The expected knowledge freely transferred by all experts to each entrepreneur during meetings is $(X-1)/2$. The relevant term from the perspective of the city, however, is the expected per entrepreneur productivity of each expert, denoted by P, with $P = \frac{1}{X}\left(\frac{X-1}{2} + \frac{1}{X+1}\right)$. There are increasing returns to X because $\frac{dP}{dX} = \frac{X^2+4-4X}{2X^2(X-2)^2} - \frac{1+2X}{X^2(X+1)^2} = \frac{1}{2X^2(X+1)^2}\left(X^2-2X-1\right) > 0$ for all integers $X > 2$. There are no increasing returns from competition when starting from $X = 1$ because if there is only one expert in the city, he bids nothing but he is hired by everyone, which is efficient compared to the case in which there are two experts who bid against each other, but with only one of them working.

city is also efficient, as expected production Y tends to its maximum level – the expected total amount of knowledge in the economy – of $(E \times X)/2$.

Intuitively, increasing returns to city size arise because interactions are cheaper and the number of potential partners is higher in larger cities. The model captures this intuition through a matching function such that each expert and entrepreneur in the city meets once, at no cost. Unsurprisingly, increasing returns at the extensive margin is a prerequisite for increasing returns at the intensive margin, and heightened competition or better matches in larger cities depend on increasing returns from the matching function. This paper formalizes and extend these ideas by combining a theory of strategic interaction – auction theory – with the fundamental properties of knowledge as an input. The idea that valuable knowledge must be given in order to demonstrate its value explains the incentive of an expert to transfer knowledge. Free reproducibility in turn leads to increasing returns, because each expert is able to transfer his knowledge to many entrepreneurs without losing it. In some sense, Arrow's property enhances the efficiency of cities. The unobservability of experts' knowledge empowers entrepreneurs to ask for free transfers during meetings. This diffusion of knowledge might not happen if the value of experts' knowledge was perfectly observable.

Another defining property of the knowledge auction lies in its ability to match each entrepreneur with the expert whose knowledge is most valuable to him. The value of an expert's knowledge is uncorrelated across entrepreneurs, so different experts, with expertise in different fields, are matched with different entrepreneurs. This is consistent with the assumption that an expert's type is not common knowledge (as it probably would be if the same expert was best for every entrepreneur) and that meetings are necessary to reveal the nature and extent of expert's knowledge. In fact, the potential for better matches between workers and firms is believed to be one of the main agglomeration forces generating cities.[18] The knowledge transfer mechanism in this paper provides a possible explanation for how entrepreneurs manage to identify more suitable workers when the pool of candidates is larger. Bigger cities not only foster more valuable informal meetings as explained above, but also more productive working relationships.[19]

To summarize, the auction framework suggests answers to the why, how and when of un-

[18]See Duranton and Puga (2004) for a review of the literature on micro-foundation for agglomeration economies

[19]The analysis of these gains is hard within the model because increasing the productivity of meeting decreases productivity at work. Note, however, that an entrepreneur gains from better matches under the assumption that he captures a share λ of a hired expert's production at work. In equilibrium, an entrepreneur can infer the type of an expert from his bid, and it is easy to show that the expected production of the best expert is larger than that of the second best. Under the assumption that $\lambda = 0$, however, 'hiring the best expert' is a good strategy for an entrepreneur only because it provides incentives for experts to transfer knowledge.

compensated knowledge transfers. For an expert, meeting an entrepreneur is an opportunity to display the extent of his knowledge, in a bid to impress an entrepreneur enough to get a job. For an entrepreneur, meeting an expert is an opportunity to receive free knowledge, and to find the best expert for a job. Market transactions, in the form of compensated knowledge transfers at the production stage of the game, only follow non-market interactions after revelation of the best expert.

2.3 *Industry examples*

The model intends to expose the general motivation behind many informal or semi-formal urban interactions. The building blocks of the model, however, are more readily apparent in industries featuring a sharp distinction between holders and users of knowledge, and between a free knowledge transfer stage and an employment stage. Examples abound in 'creative' occupations like advertising, architecture, graphic design, finance, or in job interview situations, in which the model's 'bid' is often called a 'pitch'. Studying how these industries cope with free knowledge transfers allows to determine whether the strategic incentives of the agents in the model have parallels in the real-world. In the absence of hard data, the most supportive evidence for the theory comes from industries in which free knowledge transfers are most controversial, and therefore better documented. Free transfers appear especially unfair in the presence of communication costs, which, as I show later in Section 4, can drive some experts completely out of the city.

Graphic design is such an industry, in which free transfers – called 'spec(ulative) work' or 'working on spec' – are deemed so problematic that some professional associations explicitly prohibit their members from providing free designs.[20] While a design or a logo does not possess all the strong properties of knowledge defined in this section – designs are not necessarily freely reproducible, and there is a design cost – the bans highlight the incentives of experts to give knowledge for free in a competition for jobs.

[20]The Society of Graphic Designers of Canada, for instance, prohibits its members from providing free designs, or from entering into a design contest in which only the winner is remunerated (`http://www.gdc.net/business/ethics_and_professional_practice/articles/186.php`, retrieved 10 August 2013). In the United States, the largest such professional organization, AIGA, only goes so far as stating that: "AIGA believes that professional designers should be compensated fairly for their work [...] [t]o that end, AIGA strongly encourages designers to enter into client projects with full engagement to show the value of their creative endeavor, and to be aware of all potential risks before entering into speculative work." (`http://www.aiga.org/position-spec-work/`, retrieved 10 August 2013)

In advertising and marketing, free knowledge transfers are somewhat less controversial, perhaps because of the lower cost of creating a marketing strategy or a slogan for a product. Unlike a design, a marketing advice can often be used productively even if it comes from an advisor who did not win a contract. In this case, free transfers not only generate better matches, but also more learning, as in the model.[21] This being said, there are also vocal opponents to free knowledge transfers within the advertising community. Win Without Pitching, a consulting firm for advertising agencies, voices the grievances of many agencies. An extract from The Win Without Pitching Manifesto (Enns, 2010) reads as follow: *"The forces of the creative professions are aligned against the artist. These forces pressure him to give his work away for free as a means of proving his worthiness of the assignment. Clients demand it. Advertising agencies and design firms resign themselves to it".* Indeed, competitive pressures to work for free can impose a significant burden on small players in big cities. From an efficiency perspective, one has to worry that such competition discourages individuals from acquiring knowledge, from actively developing creative ideas, or from incurring the cost of moving to a city.[22]

Finally, there is little evidence that uncompensated knowledge transfers are contentious in consulting industries, and among professionals such as lawyers, accountants, financial advisors and many other providers of business services.[23] Free knowledge transfers are part of standard networking practices. An example from the finance and banking industry is the mergers and acquisitions' (M&A) pitch, through which a firm specializing in M&A services provides a candidate firm with free information about the benefits of merging with, or acquiring another firm. Another example is 'stock pitching', an exercise to which expert practitioners have given sports-like qualities. Indeed, stock pitch competitions are popular with commerce students. Lawyers are often legally prevented from pitching free ideas to individuals or firms (to prevent 'ambulance chasing'), but most large firms have an in-house lawyer who can legally receive a pitch by another

[21] Of course, experts often consider a situation unfair precisely when an entrepreneur profits from a free advice without offering a job.

[22] These issues are mostly beyond the scope of the paper, but I do show, in Section 4, that there are always benefits from competition – increasing returns to the number of experts – when entry cost in the city are low enough.

[23] An interesting case of a free knowledge transfer in finance that *was* controversial comes from 'The Big Short', a book on the 2007-2008 financial crisis and the preceding housing bubble (Lewis, 2010) . The book tells the tale of Michael Burry, a fund manager who shares his ideas with potential investors on how to short the housing market and benefit from an eventual housing bust. Burry's idea, in retrospect, was worth billions. As it turned out, many of these potential investors did enter the trade Burry described, but never invested in Burry's fund (his guess is that they hired another 'expert', Goldman Sachs, to help). To quote Burry from the book: *"If I describe something it sounds compelling and people think they can do it themselves [...] if I don't describe it enough, it sounds scary and binary and I can't raise the capital".* This shows both Arrow's property and free reproducibility at work.

lawyer hoping to land a contract, and such free advices are relatively common (Asher, 2004). Generally, consulting industries tend to share the model's main features: knowledge is (almost) freely reproducible, and it is impossible to take back knowledge transferred for free. Also, there are often obvious benefits from hiring one consultant, ideally the best, while getting knowledge from many. For instance, an entrepreneur needs at least one lawyer to sign legal papers, or one broker to trade stocks, but there is almost unlimited potential for legal actions to take or stocks to buy.

By and large, the entrepreneurs versus experts set-up is representative of the general motivation behind urban networking. Casual meetings and professional encounters are often opportunities to tell others what and how much one knows, and to learn about and from others' knowledge. These are the essential ingredients of successful networking; of a process that can lead to a job offer directly as in the model, or indirectly through name-dropping by others, or even to recognition as a valuable partner in a reciprocal relationship. In the model, experts strive to display their knowledge because of competition for entry into a contractual relationship with an entrepreneur, but competition for entry into reciprocal relationships likely fosters very similar incentives for knowledge transfers. These incentives find clear real-world manifestations not only in the business services described above, but also in high-tech industries and clusters like Silicon Valley, which are likely the environments with the strongest agglomeration economies. In these milieus, a reputation for expertise carries a high premium, and entrepreneurs must be aware of who knows what, and of how good they are. As a result, workers in high-tech clusters experience frequent job changes, and initially accept lower wages (Freedman, 2008) while hoping for a lucrative contract after establishing their reputation.

3. Migration into an open city without sorting by types

This section and the next extend the closed city model of Section 2 to consider a country in which experts and entrepreneurs can choose to migrate to an open city. Open city models essentially add an entry cost to the model of Section 2. In this first extension, experts decide to migrate *before* learning their type, so there is no sorting of the best experts into a city (I consider a model with sorting in the second extension, in Section 4.) The main result of this section is the existence of a unique locally stable spatial equilibrium city, and the potential for cities with higher aggregate welfare, larger population and a greater proportion of experts than this equilibrium city. The auction framework also suggests a tax policy to raise a city's productivity both by attracting more experts and by enhancing their incentives to transfer knowledge for free.

There is a large number of potential experts and entrepreneurs in the countryside, whose utility is normalized to 0. As before, $N = E + X$ is the city's population, and let $z = E/X$ represent the city's composition. The number of entrepreneurs and experts in the city as a function of its population and composition, is $E = Nz/(z+1)$ and $X = N/(z+1)$. The job allocation mechanism (a first-price auction) and the production technology are as in the closed city model of Section 2. I introduce a cost of urban living C, which can be interpreted, for instance, as housing rent or the cost of driving to the city center to meet and work. C is a continuously differentiable, convex and increasing function of a city's population, with $C(0) = 0$, $C' > 0$ and $C'' > 0$ for all $N \geq 0$, and $\lim_{N \to 0} C'(N)$ is finite. $C(N)$ is the same for experts and entrepreneurs and it covers the costs of all meetings and work. An expert incurs $C(N)$ before learning her type, consistent with the idea that an expert's knowledge has a different value to different entrepreneurs, and that a meeting with an entrepreneur is necessary for an expert to discover the value of her knowledge to his particular needs.[24]

3.1 Spatial Equilibrium

When experts choose to migrate to a city before learning their type, the (sunk) cost of living does not affect their bid, or their decision to meet with all entrepreneurs. The same would be true if there was a fixed communication cost to each meeting, incurred before learning one's type. That is, the benefits of living in the city stay exactly as in Section 2. Therefore, one can use equation (5) to obtain the expected utility of an expert who migrates to a city with population N and composition z:

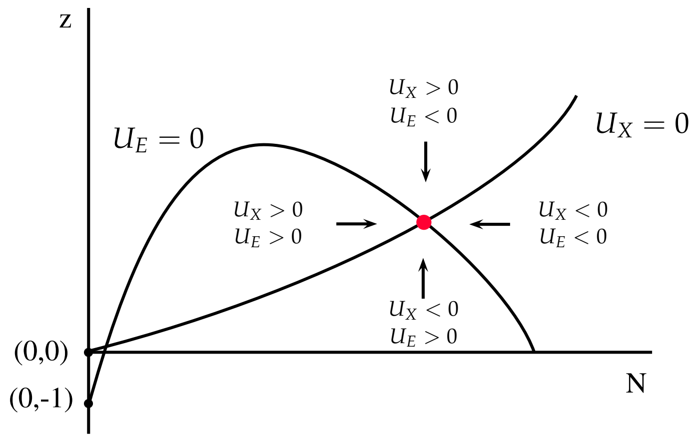
$$U_X(N,z) = \frac{E}{X(X+1)} - C(N) = \frac{z(z+1)}{N+(z+1)} - C(N), \tag{7}$$

and equation (6) to obtain the expected utility of an entrepreneur:

$$U_E(N,z) = \frac{(X-1)}{2} - C(N) = \frac{N-(z+1)}{2(z+1)} - C(N). \tag{8}$$

[24]This intuition is formalized in the assumption that $corr(k_{ij}, k_{ih}) = 0$, for all $j,h \in \{1,...,E\}$, i.e. experts are not 'good' or 'bad' for every entrepreneur, but rather their knowledge has a different value to different entrepreneurs.

Figure 1: Unique stable equilibrium in an open city



Experts and entrepreneurs only migrate if their expected utility in the city is positive, so the free-entry spatial equilibrium condition is:

$$\begin{aligned} U_E(N,z) &= 0 \\ U_X(N,z) &= 0. \end{aligned} \tag{9}$$

The easiest way to demonstrate the existence of a unique stable spatial equilibrium is to represent the indifference curves $U_E(N,z) = 0$ and $U_X(N,z) = 0$ in the (N,z) space. From equation (7), it is immediate that $\frac{\partial U_X}{\partial N} < 0$ and $\frac{\partial U_X}{\partial z} > 0$, for all $z > 0$ and $N > 0$, and therefore the indifference curve of an expert slopes upward from the origin (remind that $C(0) = 0$, so $U_X(0,0) = 0$). To determine the shape of an entrepreneur's indifference curve, I use equation (8), and isolate z from $\frac{N-(z+1)}{2(z+1)} - C(N) = 0$ to find:

$$z = \frac{N}{2C(N)+1} - 1 \tag{10}$$

$$\frac{\partial z}{\partial N} = \frac{2C(N)+1-2NC'(N)}{4(C(N)+0.5)^2}. \tag{11}$$

From equation (11), the indifference curve of an entrepreneur slopes up from $(0,-1)$, which satisfies equation (10), then down.[25] Figure 1 represents both indifference curves.

From figure 1, a locally stable equilibrium exists if the peak of an entrepreneur's indifference curve lies above the indifference curve of an expert. A sufficient condition for existence is that C

[25]To see this, note that the denominator of (11) is always positive. The limit of the numerator as $N \to 0$ is also positive, because $C(0) = 0$ and $\lim_{N\to 0} C'(N)$ is finite. As C is continuously differentiable, the indifference curve must slope up for small N. For N large enough, $\frac{\partial z}{\partial N}$ becomes negative, because $C' > 0$ and $C'' > 0$ imply that the negative term $-2NC'(N)$ grows faster than the positive term $2C(N)$ (the derivative of $NC'(N)$ is $NC''(N) + C'(N)$, and the derivative of $C(N)$ is $C'(N)$, so the assumption that $C' > 0$ and $C'' > 0$ for all $N > 0$ implies that $NC''(N) + C'(N) > C'(N)$, for all $N > 0$).

takes small enough values, meaning that if $C = \tau\widetilde{C}$, a stable equilibrium exists for τ small enough (the following proposition includes a formal statement and a proof). For instance, if $C = \tau N^2$, then it is easy to show that a stable equilibrium exists if $\tau < 0.02$.

The equilibrium is locally stable because experts and entrepreneurs move into or out of the city when their expected utility from such a move is positive. The four arrows of motion in figure 1 illustrate that through such migration, the city returns to equilibrium after any small perturbation away from it. When both $U_X > 0$ and $U_E > 0$, N must increase, because both experts and entrepreneurs benefit from moving into the city. When $U_X < 0$ and $U_E < 0$, N must decrease, because both experts and entrepreneurs benefit from moving out of the city. When $U_X < 0$ and $U_E > 0$, z must increase, because entrepreneurs gain from moving into the city while experts gain from moving out of the city. By a similar reasoning, z decreases when $U_X > 0$ and $U_E < 0$. I summarize the above results in the following proposition:

Proposition 2 *Suppose that experts move into an open city before learning their types, that the expected utility of experts and entrepreneur is given by equation* (7) *and* (8)*, that the cost of urban living is* $C = \tau\widetilde{C}$*, where* $\widetilde{C}(0) = 0$*,* $\widetilde{C}' > 0$ *and* $\widetilde{C}'' > 0$ *for all* N*,* $\lim_{N\to 0}\widetilde{C}'(N)$ *is finite, and that* E *and* X *can be any positive real numbers. Then there is a value* τ^* *such that for all* $\tau \in (0,\tau^*)$*, there exists a unique locally stable spatial equilibrium in which city composition* z *and city size* N *satisfy* $U_E(N,z) = U_X(N,z) = 0$.

Proof In Appendix D. □

If E and X must be integers, there may not be any stable equilibrium even if all other conditions of Proposition 2 are satisfied. In this case, local stability requires the existence of two integers X and E such that $U_X > 0$ and $U_E > 0$, and such that increasing X by 1 leads to $U_X < 0$, and increasing E by 1 leads to $U_E < 0$.

3.2 *City size*

Figure 1 features an area in which $U_X > 0$ and $U_E > 0$, so there exists out-of-equilibrium cities in which both experts and entrepreneurs are better-off than in equilibrium, at which $U_X = U_E = 0$. In these cities, population is always smaller than that of the equilibrium city. Excess migration is a standard result in a self-organizing equilibrium, because agents do not internalize the negative congestion externality that they impose on others, and migrate until the benefits of city living exactly offset its costs. However, a different result obtains if one can redistribute wealth among entrepreneurs and experts. With redistribution, the optimal city must only maximize aggregate welfare – total expected production minus total urban cost – and I show that the equilibrium city can be too small.

The model is not tractable enough to derive optimal or equilibrium size and composition analytically, but the following argument demonstrates the existence of cities with both population and aggregate welfare larger than that in equilibrium. First, plug $E = Nz/(z+1)$ and $X = N/(z+1)$ in the equation for total expected production Y given in Proposition 1, to obtain Y as a function of N and z:

$$Y = \frac{N^3 z + Nz^3 + 2Nz^2 + Nz}{2(z+1)^2(N+z+1)}. \tag{12}$$

In equilibrium, it must be that $z^{eq} = \{z : U_E = U_X\}$, and using equation (7) and (8), z^{eq} can be defined implicitly as $N = \sqrt{(z^{eq}+1)^2(2z^{eq}+1)}$. From this expression, z^{eq} grows like $N^{2/3}$, so the equilibrium ratio of entrepreneurs to experts increases with city size. Through tougher competition, the payoff of experts decreases with the number of other experts, so the number of entrepreneurs must increase even more to maintain experts' utility constant. Now, consider the expression in equation (12), and note that the largest exponent on N in the expression for Y represents the strength of the returns to city size. Plugging z^{eq} into equation (12) shows that Y grows like $N^{4/3}$ in equilibrium. If z were fixed, however, Y would grow like N^2. Therefore, the growth of total production Y with respect to N is faster than in equilibrium for a city with fixed – not even optimal – z. Increasing returns are weaker under self-organized migration, because of an imbalance in the ratio of entrepreneurs to experts that worsens as population increases.[26] This implies that if the urban cost function does not increase too fast, there exist cities with both population and aggregate welfare larger than that in equilibrium, and with a larger share of experts.

3.3 *City composition and taxation policy*

In the cities with positive aggregate welfare described above, experts earn a negative payoff, because N is higher and z is lower than in equilibrium (see figure 1). Redistribution is therefore necessary. The simplest policy recommendation is to reduce the cost of urban living for experts, for instance by taxing entrepreneurs and making lump-sum payments to experts.

[26] Intuitively the optimal z stays close to 1, because of the symmetry in the matching function.

There are, however, better targeted policies which act directly on the incentives of experts to transfer knowledge for free. One such policy is to subsidize the work of an expert, i.e. to offer a wage top-up w. In equilibrium, the top-up always goes to the best experts, who win the knowledge auctions. This wage subsidy is better than a lump-sum transfer, because by raising experts' valuation for a job, it provokes an increase in their knowledge bid. Appendix C shows that in a first price knowledge auction with a wage top-up w, the knowledge bid of an expert increases to $b(k,X,w) = ((X-1)/X)k + w$. If w is large enough, then experts transfer the entire value of their knowledge during a meeting.[27] In a standard first-price auction, an auctioneer cannot increase his payoff by subsidizing the winner, because the cost of the subsidy exactly offsets the increase in the winner's bid. A knowledge auction, however, is all-pay for an entrepreneur, who finds it optimal to set a wage top-up w large enough to ensure that all types of experts transfer the entire value of their knowledge during the meeting stage (see Appendix C). Subsidizing the work of experts is a good policy because it incites every expert to transfer more knowledge, even those who fail to get the job. This result relates to Lazear and Rosen (1981) 'tournament' theory, which successfully explains the presence of oversized wages at the top of a firm's hierarchy, as a means to motivate workers in the lower ranks.

3.4 Interpretation: experts vs entrepreneurs framework

The results in this section highlight the distinct role of experts and entrepreneurs. Of course, the dichotomy of the city into holders and users of knowledge is just an expository device, but it captures important features of the fabric of a city. Individuals are not equally endowed with entrepreneurial skills. Jacobs (1968) praises cities as hotbeds of entrepreneurial activities in part because densely populated areas provide entrepreneurs with more opportunities to learn. In the model, the possibility to learn from experts is exactly what motivates an entrepreneur's move to a city.

The location decision of experts has become a current public policy issue, especially following the work of Florida (2002) on the 'creative class', which he argues cities should strive to attract. Florida proposes tolerance, talent and technology as factors drawing creative types to a city, while in my model the possibility to meet entrepreneurs and to get a job drives experts to

[27]This solution assumes that the budget constraint of an expert never binds. That is, the total bid of an expert is always $b(k,X,w) = \frac{X-1}{X}k + w$, and if an expert bids more than the value of her entire knowledge, then the difference is deducted from her wage should she win the job.

migrate. Florida recommends raising the value of urban living for members of the creative class, by subsidizing amenities that they like more than other people do. Such a policy is reasonable if creative types, like experts in the model, provide positive externalities. The model suggests that targeting a subsidy to the best experts brings additional benefits, as it enhances the incentives for free knowledge transfers.[28] While theories can provide useful guidelines, there is a great need for empirical evidence on the origin and magnitude of such externalities, to aid the design of such policies.

4. Migration into an open city with sorting by types

The second extension of the model analyzes migration into an open city when experts choose to migrate *after* learning their type. In this setting, the knowledge auction becomes a first-price auction with endogenous entry cost, and only the best experts sort into the city. In what follows I describe the impact of competition for jobs, and of the cost of urban living, on the number and quality of experts living in the city, and on the size of their knowledge bid.

There is a number X_p of 'potential' experts in a country. Unlike in Section 3, the number of potential experts in the country matters, and it is not assumed to be unlimited. An expected number X_c of these potential experts migrate to an open city. The number of entrepreneurs in the city is fixed at E_c, to simplify the problem and focus on the behavior of experts, who are heterogeneous. The utility of experts outside the city is normalized to 0. Expected city population is equal to $N_c = X_c + E_c$, and the expected cost of living in the city is C, a continuously differentiable function of N_c such that $C(0) = 0$ and $C' > 0$ for all N_c. Let $c = C(N_c)/E_c$ denote, for an expert, the expected per meeting cost of living in the city. Each meeting corresponds to participation in a knowledge auction, so c is like an entry cost. Participation is never optimal if $c > 1$ (the largest possible value of knowledge), so I always assume that $c \in (0,1)$. Except for the presence of an entry cost, the knowledge auction and the production technology are exactly as in Section 2. In contrast to standard auction theory, entry costs in urban environments are endogenous, because congestion depends on the number of individuals living in the city. In a simpler, closed city version of this

[28] In the real world, in which most experts already live in different cities, place-based policies designed to attract experts to a particular city have general equilibrium effects which affects their incidence (see Kline (2010) for a recent theoretical discussion). Subsidies designed to enhance incentives for knowledge transfers, however, would still produce efficiency gains. However, I show in the next section that if experts move after learning their type, then the cost of living has a direct effect on the knowledge bid of experts, and in this case the simple policy of reducing that cost has more to recommend it.

model in which c is not endogenous, c becomes a fixed per meeting communication cost, with X_p as the total number of experts in the city, and X_c as the expected number choosing to meet with entrepreneurs.

The key assumption of this section is that experts move to a city after learning their type. This implies that the correlation between the values of an expert's knowledge for different entrepreneurs becomes important. In Section 2 and 3, the size of this correlation did not affect any of the results, but it was set to 0 to underline the knowledge auction's ability to match each entrepreneur with the best expert for his particular job. When migrating experts already know their type, however, a higher correlation leads to sharper sorting by skills. A high correlation is also consistent with the idea that experts are aware of how much their knowledge is worth. So this section restricts attention to the case in which $corr(k_{ij}, k_{ih}) = 1$, for all $j, h \in \{1, ..., E\}$ so in equilibrium the same expert is best for every entrepreneur, wins every auction and works for every entrepreneur.[29] This expert, being the best owner of a freely reproducible good, is what Rosen (1981) calls a 'Superstar'.

4.1 Number and quality of city experts

Experts simultaneously decide whether to move or not, and how much knowledge to transfer in each meeting if they move. Before solving for the equilibrium bidding function of experts, it is instructive to study their migration decision separately. So I start by analyzing how the number of experts competing for entry, X_p, and the expected per meeting cost of urban living, c, affects the number and types of experts moving to the city. I find a quantity/quality trade-off as either X_p or c varies.

Define $k^* \in (0,1)$ as a type cut-off such that experts with a value of knowledge below k^* stay in the countryside because their expected utility in the city is negative, and experts of types above k^* migrate because their expected utility in the city is positive. Experts of type k^* are indifferent. The next subsection proves the existence of k^*, but for now I assume that it exists. To find an expression for k^* as a function of c and X_p, I adapt the reasoning of Samuelson (1985). With symmetric bidding functions, a type k^* expert only wins the knowledge auctions if every other potential expert has a valuation below k^*. So the probability that a type k^* expert wins the auctions is $k^{*X_p - 1}$. Because a type k^* expert only wins when she is the sole participant, the bidding function

[29]Assuming a perfect correlation is useful to study sorting by types, but it is harder to reconcile with the idea that experts' knowledge is unobservable. Eventually one would expect the identity of the best expert to become common knowledge.

must be such that $b(k^*,X_p,c) = 0$, and her wage if she wins is equal to her full knowledge value k^*. The expected utility of this type k^* expert from any meeting is therefore equal to ${k^*}^{X_p-1}(k^* - 0) - c$, the probability that she wins, times the wage, minus the expected per meeting cost. By definition of k^*, this expected utility must equal 0 in equilibrium. Isolating k^* from ${k^*}^{X_p-1}(k^* - 0) - c = 0$, I conclude that:

$$k^* = c^{\frac{1}{X_p}}. \tag{13}$$

The expected share of potential experts moving to the city is $(1 - k^*)$, so equation (13) establishes the equilibrium relationship between X_c and X_p:

$$X_c = (1 - k^*)X_p = \left(1 - c^{\frac{1}{X_p}}\right) X_p. \tag{14}$$

From equation (13), an exogenous change in the expected per meeting cost increases the quality of experts in the city $\left(\frac{\partial k^*}{\partial c} > 0\right)$, and from equation (14) it decreases their number $\left(\frac{\partial X_c}{\partial c} < 0\right)$. The impact of competition for entry, X_p, on the expected number of city expert, X_c, depends on the derivative of equation (14) with respect to X_p:

$$\frac{dX_c}{dX_p} = (1 - k^*) - X_p \frac{dk^*}{dX_p}. \tag{15}$$

Using equation (13), I obtain an expression for $\frac{dk^*}{dX_p}$ as:

$$\frac{dk^*}{dX_p} = -\frac{c^{\frac{1}{X_p}} \ln c \frac{dc}{dX_c} \frac{dX_c}{dX_p}}{X^2}, \tag{16}$$

and finally, substituting equation (16) into equation (15) leads to:

$$\frac{dX_c}{dX_p} = \frac{1 - c^{\frac{1}{X_p}}}{1 - c^{\frac{1}{X_p}} \frac{\ln c}{X_p} \frac{dc}{dX_c}}. \tag{17}$$

The numerator of equation (17), $1 - c^{1/X_p} = 1 - k^*$, captures the direct effect of an increase in X_p, which is to raise X_c according to the proportion of experts who move. The denominator reflects both the effect of X_p on the cut-off k^*, and on the expected per meeting cost c. The next lemma is on the sign and size of $\frac{dX_c}{dX_p}$ and of $\frac{dk^*}{dX_p}$.

Lemma 2 *Suppose that the number of city entrepreneurs is fixed, that the cost of urban living satisfies $C' >$ and $C(0) = 0$, and that the per meeting costs satisfies $c < 1$ when only one expert lives in the city. Then for all values of $X_p > 1$ such that $c \in (0,1)$, it must be that as the number of potential experts increases:* 1) *the type cut-off for entry becomes higher* $\left(\frac{dk^*}{dX_p} > 0\right)$, 2) *the expected number of city experts increases* $\left(\frac{dX_c}{dX_p} > 0\right)$, *and* 3) *this increase is less than proportional to the number of potential experts* $\left(\frac{dX_c}{dX_p} < 1\right)$.

Proof In Appendix D. □

These results reveal a trade-off between the quality and the quantity of experts migrating to the city. To summarize the three main findings: First, the best experts sort into the city. This sorting by skills is consistent with empirical evidence from Combes *et al.* (2008) that more productive workers choose to live in larger cities. Second, an exogenous increase in the expected per meeting cost of urban living generates an increase in the average quality of experts in the city, and a decrease in their expected number. Third, an increase in the number of potential experts (competition) leads to an increase in the average quality of experts in the city, a decrease in the proportion of experts who move, and an increase their expected number (Lemma 2). Overall, this analysis highlights the mixed blessing of urban attributes like low communication cost, which decreases the quality of experts in the city, or high competition for entry, which lowers the proportion of experts who migrate and become productive.

4.2 Bidding function

The complete equilibrium strategy of experts is the solution to a first-price auction with an entry cost.[30] In this section, the value of expert i's knowledge is the same for every entrepreneur, so she enters the city if her expected utility from these – equivalent – meetings is positive. For each $k_i \in [0,1]$, she can stay in the countryside where her utility is 0, or enter the city and submit a bid $b_i \geq 0$ in each meeting, that maximizes the probability that she wins a job, times her wage, minus the entry cost. In a symmetric equilibrium, all other experts use a bidding function b and expert i solves:

$$\max_{stay,enter}\{0, \max_{b_i} prob\left(b_i > \max_{l \in X_c \setminus \{i\}} b(k_l,X_p,c)\right) * (k_i - b_i) - c\}. \tag{18}$$

Lemma 3 *For an expert, a meeting is equivalent to participation in a first-price auction with an entry cost that requires solving equation* (18)*. There exists a unique symmetric Bayesian Nash equilibrium with a bidding function that is strictly increasing in k and $b(k,X_p,c) = 0$ for $k \in [0,k^*]$. In this equilibrium, $k^* = c^{1/X_p}$, experts of type $k < k^*$ do not move to the city, experts of type $k = k^*$ are indifferent between not moving and moving with a bid equal to $b(k^*,X_p,c) = 0$, and experts of type $k > k^*$ move to the city and bid $b(k,X_p,c) = \frac{X_p-1}{X_p}\left(k - \frac{c}{k^{X_p-1}}\right)$.*

[30]The endogeneity of c does not matter when solving for the bidding function of experts. It needs to be taken into account when analyzing, for instance, how exogenous changes in X_p affect the bid.

Proof In Appendix D. □

An important property of the bidding function is that it decreases linearly with c. That is, a unique feature of this section's model is that aggregate production depends directly on the cost of urban living, because it affects the willingness of experts to transfer knowledge for free. This is relevant to the design of policies discussed in Section 3: the simple policy of lowering the cost of entry for all experts gains an edge, because reducing c also increases knowledge bids.

The effect of an increase in the number of experts competing for entry into the city is not to unambiguously increase knowledge bids, as in the model without an entry cost of Section 2. $\frac{\partial b}{\partial X_p}$ is positive for small X_p, but as X_p increases to infinity the bid of each type of experts drops to zero.[31] For any finite value of X_p, however, a low enough value of c always gives rise to increasing returns from competition between experts. To see this, remind that $c = C(N_c)/E_c$ is a function, so a formalization of the condition that c be small enough is to define $c = \tau(C(N_c)/E_c)$, and to consider small values of the parameter τ. Defining P_p as the per entrepreneur productivity of a potential expert, Section 2 demonstrates that $\frac{dP_p}{dX_p} > 0$ when $\tau = 0$, and it is easy to show that $\frac{dP_p}{dX_p}$ is continuous with respect to τ, which implies that $\frac{dP_p}{dX_p} > 0$ for τ small enough. That is, increasing returns to the number of potential experts in the country arise when the expected per meeting cost is low enough. There is little empirical evidence of increasing returns to population at the country level, but even negligible returns to the number of potential experts can be magnified into sizable returns to the number of city experts, that are observed in practice. This happens because experts staying in the countryside are not productive, while competition spurs both higher bids and sharper sorting of experts moving into the city.[32]

Interpretation: effect of communication costs on knowledge diffusion

The knowledge auction with an entry cost uncovers important trade-offs between the number and the quality of urban interactions. The downtowns of large cities are expensive places to live in, and face-to-face interactions are often costlier than other modes of communication. The impact of entry costs on the quality of participants in urban meetings is reminiscent of Storper and Venables (2004)

[31] $\frac{\partial b(k,X_p,c)}{\partial X_p} = \left(k - \frac{c}{k^{X_p-1}}\right)\frac{1}{X_p^2} - \frac{X_p-1}{X_p}\left(\frac{\frac{dc}{dX_c}\frac{dX_c}{dX_p}k^{X_p-1} - ck^{X_p-1}\ln k}{k^{2(X_p-1)}}\right)$ is positive at $X_p = 1$ whenever the first term $k - \frac{c}{k^{X_p-1}}$ is positive, which it is for all experts who move to a city (i.e. with types $k > k^* = c^{\frac{1}{X_p}}$). The function $\frac{\partial b}{\partial X_p}$ is continuous with respect to X_p, so it also takes positive values for small enough $X_p > 1$.

[32] In a dynamic setting with many cities, Rossi-Hansberg and Wright (2007) show that a balanced growth path at the country level is consistent with increasing returns at the city level.

quip about face-to-face interactions, that 'the medium *is* the message'. In a knowledge auction the message itself matters, but there are benefits from high entry costs, which encourage stricter sorting of the best experts into meetings. Among mechanisms for screening valuable communication partners, the leading alternative to voluntary sorting by willingness to pay is the design of reputation systems. Intuitively, reputation formation mechanisms become essential when communication costs are low. For instance, such mechanisms are key prerequisite for productive use of the Internet. In general, sorting and reputation formation are not mutually exclusive and can be part of the same process. If experts in the model were able to develop a reputation for being knowledgeable, they would be even better disposed towards costly knowledge transfer meetings.

Of course, lower communication costs also have advantages. In the model, a reduction in the per meeting cost directly increases the size of experts' knowledge transfers. This demonstrates how low communication costs can enhance incentives to disseminate knowledge. The model is mute on knowledge creation and technological progress, but the diffusion of knowledge is often a precondition to its creation. While many authors have rightly emphasized the salience of social norms and diversity for policies aiming at faster knowledge growth,[33] lowering the costs and augmenting the rewards from learning about others' ideas are also obvious targets.

5. Conclusion

Urban environments offer opportunities to learn and network, which take center stage in recent thinking about the purpose and the future of cities. Starting from the key properties of knowledge as an input in production, I model knowledge transfers as bids in first-price auctions for jobs. The knowledge auction illustrates why knowledge sometimes 'spills' in non-market interactions before being bought and sold in markets. Endogenous agglomeration economies arise because of the greater number of meeting partners in urban areas, and heightened competition for jobs that enhances incentives for knowledge transfers. The model is too stylized to offer clear-cut policy recommendations, and it excludes the impact of reciprocity and diversity. However, it can inform discussions and empirical work on the origin of knowledge spillovers, and on how the cost of

[33] Saxenian's (1994) comparative analysis of the technology clusters in Silicon Valley and Route 128 highlights social norms favoring knowledge flows as drivers of Silicon Valley's success. A theoretical literature emphasizes the importance of knowledge heterogeneity, see for instance Berliant, Reed III, and Wang (2006). Empirical studies also explore the impact of diversity on group decision making, for instance, West and Dellanab (2009) find that both ability diversity and cognitive diversity reduce group decision errors. Of course, my model overlooks a significant component of the productivity gains from agglomeration, by ignoring the complementarities between the skills of different knowledge workers. Through such complementarities, an expert could derive direct benefits from a location near other experts.

meeting and the number of potential competitors and partners affects an individual's decision to move to a city and transfer her knowledge.

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Appendix A. Auctions with reservation price: requiring a minimum knowledge bid

This appendix extends the knowledge auction to allow entrepreneurs to set a minimum knowledge bid. I show that whenever at least three experts participates in the auction, entrepreneurs prefer not to require a minimum knowledge bid.

Suppose that an entrepreneur asks that any expert wanting to participate in the knowledge auction bids at least an amount r of knowledge. That is, r is the equivalent of a reservation price in a standard auction set up. Everything else is just as in Section 2, so the model satisfies all the conditions in Myerson (1981), who shows that a symmetric, increasing Bayesian Nash equilibrium bidding function exists for a class of auctions with reservation price, and also that a first price auction with a reservation price is optimal in a standard set up in which the seller only receives the winning bid. Given the existence of an equilibrium, an expert's bidding function must be $b(k,r,X) = 0$ if $k < r$, and

$$b(k,r,X) = \mathbb{E}\left(max\{k_{(X-1)},r\}|k_{(X-1)} < k\right) \text{ if } k \geq r, \tag{A1}$$

where $k_{(X-1)}$ is the highest order statistics of $X-1$ draws from the *i.i.d.* $U[0,1]$ distribution, with probability distribution function $f(k_{(X-1)}) = (X-1)k_{(X-1)}^{X-2}$. So we can write:

$$b(k,r,X) = \frac{1}{k^{X-1}} \int_0^r rf(k_{(X-1)})dk_{(X-1)} + \int_r^k kf(k_{(X-1)})dk_{(X-1)} \text{ if } k \geq r,$$

and solve for:

$$b(k,r,X) = \frac{X-1}{X}k + \frac{r^X}{Xk^{X-1}} \text{ if if } k \geq r.$$

To find the optimal reservation price r^*, it is sufficient to maximize the expected value of knowledge transferred by an expert, so that:

$$r^* = \underset{r}{\text{argmax}} \int_r^1 b(z,r,X)dz.$$

Solving the integral in the maximization problem leads to:

$$\int_r^1 b(z,r,X)dz = \int_r^1 \left(\frac{X-1}{X}z + \frac{r^X}{Xz^{X-1}}\right) dz = \frac{X-1}{2X}(1-r^2) + \frac{1}{X(2-X)}(r^X - r^2).$$

Taking a derivative of the expression above with respect to r, I obtain:

$$\frac{-(X-1)2r}{2X} + \frac{1}{X(2-X)}\left(Xr^{X-1} - 2r\right) = -2r\left(\frac{3-X}{2(2-X)}\right) + \frac{r^{X-1}}{(2-X)}.$$

This derivative takes negative values for all X larger than some number between 2 and 3, so if there are at least 3 bidders, entrepreneur optimally set $r^* = 0$. Remind that a knowledge auction is all-pay from the perspective of an entrepreneur, so a reservation price is not desirable because it deters low type experts from participating in the auction.

Appendix B. Discriminatory auctions: hiring more than one expert

This appendix extends the knowledge auction to allow entrepreneurs to hire more than one expert. I provide numerical evidence that the first-price auction is optimal, because hiring many experts reduces their knowledge bids.

Suppose that entrepreneurs offer a number Q of jobs, to the experts who bid the highest amount of knowledge during meetings. The wage of experts is equal to the value of their remaining knowledge, exactly as in Section 2. Therefore, experts participate in a kind of multi-unit auction called a discriminatory auction, because different bidders 'pay' a different price for a unit of the good. Given that experts have a concave utility function, Harris and Raviv (1981) show that there exists a symmetric, increasing Bayesian Nash equilibrium bidding function, given by:

$$b(k,X,Q) = \mathbb{E}\left(k_{(X-1),(X-Q)} | k_{(X-1),(X-Q)} < k\right), \tag{A2}$$

where $k_{(X-1),(X-Q)}$ is the $(X-Q)^{th}$ order statistics from $X-1$ draws of the *i.i.d.* $U[0,1]$ distribution. Cox, Smith, and Walker (1985) show that (A2) can be written in terms of Beta integrals, to obtain:

$$b(k,X,Q) = \frac{\int_0^k Y^{X-Q}(1-Y)^{Q-1}dY}{\int_0^k Y^{X-Q-1}(1-Y)^{Q-1}dY} = \frac{\sum_{i=1}^{Q} \frac{(-1)^{i-1}k^{X-Q+i}(Q-1)!}{(X-Q+i)(i-1)!(Q-i)!}}{\sum_{i=1}^{Q} \frac{(-1)^{i-1}k^{X-Q+i-1}(Q-1)!}{(X-Q+i-1)(i-1)!(Q-i)!}}. \tag{A3}$$

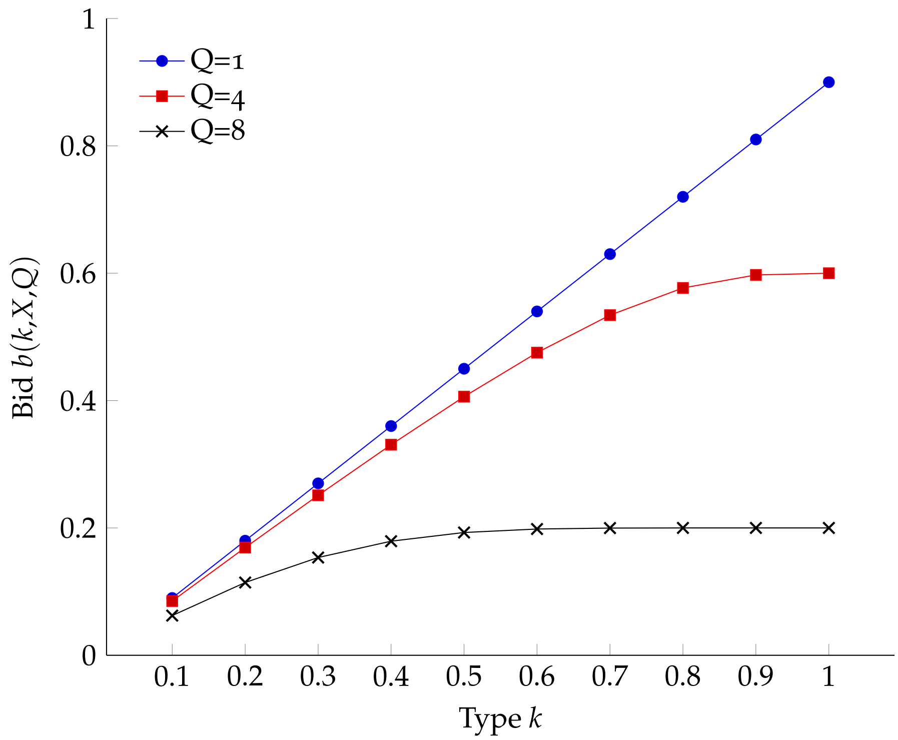
The expression in equation (A3) is not tractable enough to provide analytical results. Figure 2 plots the bid of each expert as a function of her type, for different values of the number of job offered Q. For each type of expert, the bids in Figure 2 decrease with the number of job offers.[34] This suggests that an entrepreneur prefers to hire the minimum number of experts, $Q = 1$, which maximizes the knowledge bid of each type of expert at the meeting stage of the game. This result depends on the assumption that entrepreneurs do not get a share λ – or get a small enough share – of the surplus created by working experts. As λ tends to one, it becomes optimal for entrepreneurs to hire all experts, and to capture the entire value of their knowledge at the production stage of the game.

Appendix C. Auctions with transfer payments: offering a wage top-up

This appendix extends the model to let entrepreneurs make transfer payments to experts, outside of the wage setting mechanism of section 2. I restrict attention to transfers going to experts who win a knowledge auction, which are equivalent to a wage top-up, denoted by w. I show that an entrepreneur gains from setting $w > 0$, and that any w large enough to ensure that all types of expert bid the entire value of their knowledge is optimal.

[34]Inspection of equation (A2) strongly suggests that the plots in Figure 2 are representative of a general result. The reason is, for any range of values $[0,k]$, the probability distribution of the $(X-Q)^{th}$ highest draw from the $U[0,1]$ distribution must put relatively more weight on high values of k than the probability distribution of the $(X-(Q+1))^{th}$ highest draw. It follows that the conditional expectation in equation (A2) must indeed decreases with Q. To the best of my knowledge, this simple intuition does not correspond to a known result in probability theory.

Figure 2: Bidding function for different number of hired experts.



Notes: Q is the number of job offered by an entrepreneur. All three bidding function are computed at X=10.

Suppose that expert i meets an entrepreneur j offering a wage top-up w_j, and that all other experts use a bidding function b. Then, this expert submits a bid $b_i \geq 0$ that solves:

$$\max_{b_i} prob \left(b_i > \max_{l \in \{1,\ldots,X\} \setminus \{i\}} b(k_{lj}, X, w_j) \right) \times (k_{ij} + w_j - b_i). \tag{A4}$$

This expression is similar to equation (2) governing experts' behavior in the basic first-price auction of Section 2, aside from the extra w_j in the payoff. Equation (A4) implicitly assumes that an expert does not face a binding budget constraint. That is, whenever an expert bids more than the entire value of her knowledge, any part of the bid that exceeds her knowledge's value is deducted from her wage should she get the job. It is now trivial to modify lemma 3 and prove that the bidding function becomes $b(k,X,w) = \frac{X-1}{X}k + w$ (I removed subscripts to improve legibility).

The optimal w is that which maximizes U_E, the expected utility of an entrepreneur, which is equal to:

$$U_E = X \left(\int_0^{wX} z dz + \int_{wX}^{1} \left(\frac{X-1}{X} z + w \right) dz \right) + \int_0^{wx} \left(w - \frac{z}{X} \right) X z^{X-1} dz - w \tag{A5}$$

The first of the three terms in equation (A5) is the expected value of the total knowledge transferred by all experts during meetings. The first integral in the parenthesis of the first term is the expected value of the bids for experts with types $((X-1)/X)k + w > k$ who bid the entire value of their knowledge at the meeting stage.[35] The second integral in the parenthesis is the expected value of the bids for experts who have enough knowledge to transfer $((X-1)/X)k + w$ at the meeting stage. The second term in equation (A5) is the expected deduction from the wage of an expert who won the auction but bid all her knowledge at the meeting stage, and owes the difference $((X-1)/X)k + w - k = w - \frac{k}{X}$ to the entrepreneur.[36] The third term is minus the wage top-up. Note that equation (A5) is only correct over the domain $w \in [0,1/X]$. For w larger that $1/X$, all types of experts bid their entire knowledge at the meeting stage.

To find the value of w that maximizes U_E, I solve the integrals in equation (A4) to obtain:

$$U_E = \frac{X-1}{2} + w(X-1) - w^2\frac{X^2}{2} + \frac{(wX)^{X+1}}{X(X+1)}$$

and take a derivative with respect to w to find:

$$\frac{\partial U_E}{\partial w} = (X-1) - wX^2 + (wX)^X.$$

This derivative is positive at $w = 0$, and it decreases with w until it reaches 0 at $w = 1/X$. Therefore, over the domain $w \in [0,1/X]$, the expected utility of an entrepreneur reaches a maximum at $w = 1/X$, which is a wage top-up just high enough to ensure that all types of experts bid the entire value of their knowledge. At $w = 1/X$, the expected payoff of an entrepreneur is $\frac{X}{2} - \frac{1}{1+X}$, and it is easy to show that it stays constant for all $w > 1/X$. I conclude that entrepreneur chooses a wage-top up large enough to motivate all types of experts to transfer all their knowledge for free at the meeting stage.

Appendix D. Proofs

Proof of Proposition 2

I show that as the cost of urban living $C = \tau\widetilde{C}$ becomes smaller (i.e. as τ decreases), both indifference curves must cross as in figure 1. First, recall from equation (8) that $U_E = \frac{N-(z+1)}{2(z+1)} - C(N)$, so that $\frac{\partial U_E}{\partial z} < 0$. Therefore if $C(N)$ decreases for every value of $N > 0$, then z must increase for every $N > 0$ to maintain an entrepreneur's utility constant. So as τ decreases, the curve $U_E = 0$ shifts

[35] To compute the boundaries of the integral, note that wX solves $((X-1)/X)k + w = k$

[36] Remind that $f(k_{(X)}) = Xk_{(X)}^{X-1}$ is the probability distribution function of the highest order statistic from X draws of the *i.i.d.* $U[0,1]$ distribution.

up at every $N > 0$. Similarly, from equation (7), $U_X = \frac{z(z+1)}{N+(z+1)} - C(N)$, so that $\frac{\partial U_X}{\partial z} > 0$. Again, if $C(N)$ decreases for every value of $N > 0$, then z must decrease for every $N > 0$ to maintain an expert's utility constant. So as τ decreases, the curve $U_X = 0$ shifts down at every $N > 0$.

I show in the text that $U_X = 0$ slopes up from $(0,0)$ in the (N,z) space, and that $U_E = 0$ slopes up from $(0, -1)$, then down. I conclude that if a decrease in τ shifts up $U_E = 0$ and shifts down $U_X = 0$ for every $N > 0$, then as τ gets close enough to 0 the two curves necessarily cross. The argument for local stability of the equilibrium in which $U_E = 0$ crosses $U_X = 0$ from above is presented in the text. Note that in this case there will also be another - unstable - equilibrium with smaller N and z, as $U_E = 0$ crosses $U_X = 0$ from below.

Proof of Lemma 2

If $c \in (0,1)$ then the numerator in equation (17) is between 0 and 1, and the denominator is larger than 1 because $\frac{dc}{dX_c} > 0$. I conclude that $0 < \frac{dX_c}{dX_p} < 1$, which proves part 2) and 3) of the lemma. From equation (16), it is immediate that $\frac{dk^*}{dX_p} > 0$ if $\frac{dX_c}{dX_p} > 0$, which proves part 1).

Proof of Lemma 3

(To prove Lemma 1, set c equals to 0.) I first derive the bidding function. The derivation is based on the idea that there are no profitable deviation in a Nash equilibrium. Suppose that all experts moving to the city bid according to the same function b of their types, except for an expert of type k who deviates and bids like an expert of type s. This expert's expected utility is $u(s,k) = s^{X_p-1}(k - b(s,X_p,c)) - c$. The probability that she wins is s^{X_p-1}, because the best expert wins and bidding functions are symmetric and strictly increasing. If b is an Nash equilibrium bidding function, then $u(s,k)$ must reach its maximum at $s = k$. The first order condition is:

$$\frac{\partial u(s,k)}{\partial s} = (X_p - 1)s^{X_p-2}(k - b(s,X_p,c)) - s^{X_p-1}\frac{\partial b(s,X_p,c)}{\partial s} = 0.$$

Setting $s = k$ leads to:

$$(X_p - 1)k^{X_p-2}b(k,X_p,c) + k^{X_p-1}\frac{\partial b(k,X_p,c)}{\partial k} = (X_p - 1)k^{X_p-1},$$

which can be rearranged as:

$$\frac{\partial \left(k^{X_p-1}b(k,X_p,c)\right)}{\partial k} = (X_p - 1)k^{X_p-1}.$$

Integrating on both sides (types below $k^* = c^{\frac{1}{X_p}}$ bid 0, which sets the lower bound of integration) leads to:

$$k^{X_p-1}b(k,X_p,c) = (X_p - 1)\int_{k^*=c^{\frac{1}{X_p}}}^{k} z^{X_p-1}dz.$$

After solving the integral, one can isolate the equilibrium bidding function:

$$b(k,X_p,c) = \frac{X_p - 1}{X_p}\left(k - \frac{c}{k^{X_p-1}}\right).$$

I now prove that the strategy outlined in the lemma is the unique symmetric Bayesian Nash equilibrium. The expected utility of an expert of type k who moves to the city but deviates by bidding like an expert of type s is $u(s,k) = s^{X_p-1}(k - b(s,X_p,c)) - c = s^{X_p-1}\left(k - \frac{X_p-1}{X_p}\left(s - \frac{c}{s^{X_p-1}}\right)\right) - c$. The first derivative of this function with respect to s is $s^{X-2}\left(X_p - 1\right)(k - s)$, which is positive for $s < k$, negative for $s > k$, and equal to 0 at $s = k$. So for an expert moving to the city, there are no profitable deviations from bidding $b(k,X_p,c) = \frac{X_p-1}{X_p}\left(k - \frac{c}{k^{X_p-1}}\right)$. To find the types of experts who find it profitable to move, replace s by k in $u(s,k)$, to obtain type k's maximum attainable utility in the city: $k^{X_p-1}\left(k - \frac{X_p-1}{X_p}\left(k - \frac{c}{k^{X_p-1}}\right)\right) - c = \frac{k^{X_p}-c}{X_p}$. From this expression, I conclude that an expert is indifferent between moving or not for $k = k^* = c^{\frac{1}{X_p}}$ (because $(k^{X_p} - c)/X_p = 0$), prefers to move at $k > k^*$ (because $(k^{X_p} - c)/X_p > 0$) and not to move at $k < k^*$ (because $(k^{X_p} - c)/X_p < 0$).