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Urban Sprawl and the Property Tax

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Abstract

This paper explores the connection between the property tax and urban sprawl. While the tax's depressing effect on improvements reduces population density, spurring the spatial expansion of cities, a countervailing effect from lower dwelling sizes may dominate, raising densities and making cities smaller. The analysis shows that this latter outcome is guaranteed under CES preferences when the elasticity of substitution σ is high. But numerical results for the Leontief case (where σ is zero) suggest that the property tax encourages urban sprawl when substitution between housing and other goods is low. Thus, the distortions generated by the property tax may include inefficient spatial expansion of cities, suggesting the tax may belong on the list of causal factors identified by critics of urban sprawl.

Keywords: urban sprawl, property tax

JEL Code: H71, R10, R14

1. Introduction

Urban sprawl has become an important policy issue in the U.S. in recent years. While sprawl is partly a descriptive term, referring to the spatial expansion of cities, it also has a pejorative connotation, implying the normative judgement that urban spatial growth is excessive. Thus, critics of urban sprawl argue that long commutes, traffic congestion, and the rapid conversion of agricultural land are evidence that American cities have expanded too much. This belief has spawned numerous policy measures at the state and local levels designed to limit the spatial expansion of U.S. cities (see Brueckner, 2001b for details).

Urban economic theory tells us that the spatial growth of cities is a result of several fundamental forces. These include expansion of the U.S. population as well as the rise in household incomes, which spurs the demand for land by encouraging consumption of larger houses. In addition, urban spatial expansion is a natural consequence of heavy investment in transportation infrastructure (mainly freeways), which eases commuting and thus encourages suburban living.

While urban growth purely in response to these forces cannot be faulted as inefficient, criticism of the growth process is justified if the operation of the fundamental forces is distorted by market failure. The literature has, in fact, identified several market failures that might cause such a distortion, potentially leading to excessive spatial expansion of cities.[1]

These arise from the failure to internalize two externalities: the positive externality from open space around cities; and the negative externality associated with road congestion. Internalizing the open-space externality via a development tax would slow urban growth, and imposing congestion tolls to address the second externality would raise the private cost of commuting, leading to shorter commutes and more-compact cities.[2]

In addition, a fiscal distortion created by underpricing of urban infrastructure may also encourage excessive spatial growth of cities, as shown by Brueckner (1997). The problem is that local tax systems usually require developers to pay only a fraction of the infrastructure costs associated with their projects, which makes development look artificially cheap and encourages urban expansion. The remedy is to levy "impact fees," where developers are charged for the full cost of infrastructure.

The present paper investigates another possible source of urban sprawl arising from the institutions of local public finance. In particular, the paper explores the connection between urban spatial expansion and the property tax. At first glance, there would appear to be no obvious link between property taxation and sprawl. However, the connection becomes clear when the lessons of the long-standing debate on land taxation and its virtues are recalled. To see this point, note first that the property tax can be viewed as a tax levied at equal rates on both the land and capital embodied in structures. While the land portion of the property tax has no effect on resource allocation in a static setting, the tax on capital (i.e., improvements) tends to lower the equilibrium level of improvements chosen by the developer. Thus, land is developed less intensively under property taxation than under a pure land tax, where the rate on improvements is set at zero. This conclusion has been derived and discussed many times in the literature.[3]

To see the implications of this conclusion for urban sprawl, note that in the case of residential structures, a lower level of improvements per acre means that developers construct shorter buildings, containing less housing floor space per acre of land. If the size of dwellings within each building were to remain constant, then a shorter building height implies a decline in population density, with fewer households fitting on each acre of land. But if the city must accommodate a fixed population, lower densities mean that it must take up more space. Thus, by reducing the intensity of land development, the property tax would appear to encourage urban sprawl. This effect, which was analyzed in an earlier paper by Brueckner (2001a), suggests that the distortions generated by the property tax may include excessive spatial expansion of cities.

It should be noted that, since the property tax has been an important revenue source in the U.S. for decades, it cannot be viewed as a culprit in the recent urban-growth explosion that has prompted public concern. However, the previous discussion suggests that the tax may exacerbate underlying trends by depressing the intensity of land development.

Despite this compelling link between property taxation and urban expansion, there exists a countervailing effect that runs in the opposite direction. This effect operates through the property tax's impact on dwelling sizes. Because the tax on land and structures is partly shifted forward to consumers, leading to a higher price of housing floor space, dwelling size decreases in response. But smaller dwellings imply an *increase* in population density, which tends to offset the density decline caused by lower improvements. This density increase, in turn, puts downward pressure on the city's spatial size. The upshot is that in a full analysis, the net effect of the property tax on the spatial sizes of cities is ambiguous. If the

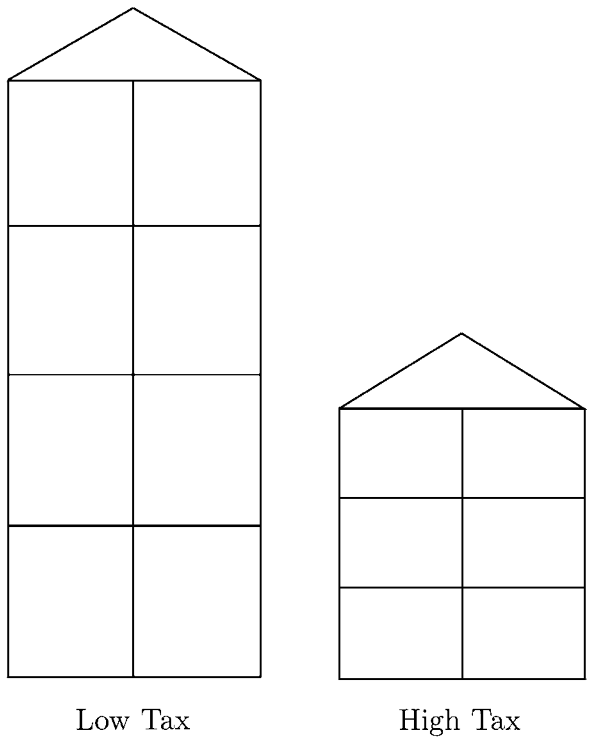

Figure 1. Improvements and dwelling-size effects.

improvement effect dominates, then the property tax encourages urban sprawl. But if the dwelling-size effect is more important, then the property tax actually helps retard sprawl by encouraging smaller cities.

These countervailing effects are illustrated in Figure 1. The left side of the Figure shows a tall building divided into large dwellings, corresponding to the case of a low property-tax rate. By contrast, the right side shows a short building divided into small dwellings, corresponding to the case of a high tax rate. Because the improvements and dwelling-size effects work in opposite directions, dwellings per unit of land, which gives population density, could be higher in either case. However, the Figure shows a situation where the improvements effect dominates, yielding lower density (and thus a spatially larger city) when the property-tax rate is high.

The purpose of the present paper is to provide a formal analysis that demonstrates these points, exploring the link between the property tax and urban sprawl. In addition to showing the separate effects of the tax on improvements and dwelling size in a spatial context, the analysis attempts to gauge the likely direction of the tax's net effect on the city's land area. Since only limited analytical results are available regarding this net effect, the paper also presents numerical examples.

Because the policy focus on sprawl is relatively recent, previous authors had little incentive to provide a full treatment of the connection between the property tax and the spatial sizes of cities. Carlton (1981) and LeRoy (1976) analyze certain effects of the property tax in a spatial context, without considering its impact on urban expansion. Sullivan (1985) presents simulation analysis of the spatial effect of the property tax, but his use of a complex model that includes both business and residential property makes it hard to see the operation of the forces sketched above. Mills (1998) investigates the spatial effect of the property tax in a business-oriented model with no residential land, so that the above reasoning does not apply. In a more-closely related paper, Arnott and MacKinnon (1977) present a simulation analysis of the effects of the property tax in a purely residential model, focusing on a host of incidence questions (one of which is the spatial-size issue). The main version of their model is apparently equivalent to the one used below despite a different presentation, and it generates spatial shrinkage of the city in response to an increase in the property-tax rate. However, Arnott and MacKinnon provide little discussion of this effect and no analysis highlighting the forces involved. As will be shown below, their model parameterization represents a special case in a more general framework where both shrinkage and expansion of the city are possible responses to an increase in the property-tax rate. As a result, the present analysis complements their work.

The plan of the paper is as follows. Section 2 presents the model, and Section 3 carries out comparative-static analysis showing the effect of the property-tax rate on the spatial size of the city. Section 4 presents numerical examples, which are designed to explore the net impact of the effects identified in the analysis. Section 5 discusses efficiency issues, and Section 6 offers conclusions.

2. The Model

2.1. Standard Results on Property Taxation and Improvements

It is useful to begin by developing the housing production side of the model. This development will allow illustration of received wisdom on the effect of the property tax on improvements in a partial-equilibrium, nonspatial context. With this background, the discussion then turns to presentation of the general equilibrium spatial analysis.

In the model, housing is produced by combining capital and land. Since the production function exhibits constant returns to scale, the analysis can focus on the choice of capital per acre of land, or "improvements" per acre. Letting S denote improvements per acre, housing output per acre of land is given by the function $h(S)$, which is increasing and strictly concave, reflecting diminishing returns to improvements (thus, $h' > 0$ and $h'' < 0$). Note that improvements (per acre) is an index of building height, which registers the intensity of land development, while housing output is measured in units of floor space. Thus, an increase in improvements leads to a taller building, which contains more housing floor space than the original building.

Since the cost per unit of improvements equals i, the housing developer's improvement cost per acre equals iS (i is exogenous). With land cost per acre equal to land rent, denoted r, the developer's total cost per acre is given by $iS + r$ in the absence of any taxes. To introduce property taxation, it is convenient to represent the property tax as a tax levied at equal rates on both land and improvements. An algebraically equivalent formulation portrays the property tax as a tax on the developer's housing output. Adopting the first approach, and letting θ denote the property-tax rate, the developer's total cost inclusive of the tax is given by $(1+\theta)(iS+r)$. Letting p denote the rental price per unit of housing floor space, the developer's profit per acre is then equal to $ph(S)-(1+\theta)(iS+r)$.

The property tax's effect on the intensity of development can be derived by computing the impact of an increase in the tax rate θ on the level of improvements S chosen by the developer. In the previous literature, this exercise is usually carried out in a partial-equilibrium setting where the housing price p is fixed exogenously. Using the above profit expression, the first-order condition for choice of S is $ph'(S)=(1+\theta)i$. When the tax rate θ increases with p held fixed, the marginal-cost term on the RHS rises, and the developer responds by reducing the level of improvements.

To avoid distorting the choice of improvements, the local government could abandon the property tax and switch to a land tax. Under such a tax, improvements are not taxed at all, with land taxed at a rate τ. The developer's profit per acre in this case is $ph(S)-iS-(1+\tau)r$, and the first-order condition for choice of S is $ph'(S)=i$. Since this condition does not involve τ, it follows that the level of improvements is unaffected by the tax rate on land.

To see the connection between this analysis and urban sprawl, observe that population density is given by housing floor space per acre of land, $h(S)$, divided by housing floor space per person, denoted q (this is dwelling size). The resulting ratio, $h(S)/q$, gives dwellings (and hence people) per acre of land, or population density (recall Figure 1). Given the above discussion, the decline in improvements as θ increases leads to a reduction in housing output $h(S)$ per acre. Unless this effect is offset by a lower dwelling size q, population density falls with θ. As explained above, this lower density should lead to a spatial expansion of the city.

In order to explore these effects, the model must be modified to allow the city's land area to be endogenouly determined. The easiest way to achieve this endogeneity is to suppose that the city's residents commute to work in a central business district (CBD). With CBD access valued, land rent then falls as distance to the center increases, and this decline imposes a natural limit on the size of the city. When property taxation is added to this type of model, the effects on population density are more complex than those sketched above. These effects are analyzed in the subsequent discussion.

2.2. The Spatial Model

The spatial model is developed in standard fashion, following Brueckner's (1987) extension of the analysis of Wheaton (1974). Let x denote the distance from a consumer's residence to the CBD, and suppose that commuting cost per period from distance x is given by tx, where $t>0$. Letting y denote the common income per period earned at the CBD by all urban residents, disposable income for an individual living at distance x is then equal to $y-tx$. Letting c denote consumption of the numeraire nonhousing good, and recalling that q is dwelling size, the budget constraint of a household living at distance x is then $c+pq=y-tx$, where p again is the rental price per square foot of floor space. Finally, let $v(c,q)$ denote the urban residents' common, well-behaved utility function.

At each location, individuals choose c and q to maximize utility, and because all residents are identical, they must achieve the same utility in equilibrium. As a result, the first-order condition $v_q/v_c = p$ and the uniform-utility condition $v(y - tx - pq, q) = u$ must hold at each location, where u is the urban utility level and where c has been eliminated via the budget constraint (subscripts denote partial derivatives). These conditions can be solved simultaneously to determine p and q as functions of parameters. For the present analysis, only the parameters x and u are of interest, allowing the solutions to be written $p(x, u)$ and $q(x, u)$.

Note that while the utility level u is ultimately endogenous, it is viewed as parametric at this stage in the analysis. Thus, the uniform-utility condition fixes the level of the indifference curve, and the first-order condition says that the budget line must be tangent to this fixed curve. With the budget line's c intercept fixed at $y - tx$, these requirements then determine the slope of the line $(-p)$ as well as the q value at the tangency as functions of location x and the utility level u.

Brueckner (1987) and other authors show that p and q vary with the parameters in the following fashion:

$$p_x < 0,\ p_u < 0; \qquad q_x > 0,\ q_u > 0. \tag{1}$$

Because disposable income falls with x, the housing price must fall as well to maintain a constant utility level. Thus, housing is cheaper on a per square foot basis far from the city center. In response to this price decline, dwelling size rises with x, with the tangency point moving along the fixed indifference curve in the direction of a higher q. As u rises, the consumer must reach a higher indifference curve, and this requires an outward rotation of the budget line and thus a lower absolute slope p. Note that the decline in p occurs at all distances. Provided that housing is a normal good, this rotation of the budget line leads to a new tangency point with a higher value of q. Thus, dwelling sizes rise at all locations as utility increases.

Although the following analysis relies mainly on the signs of the derivatives in (1), the expression for p_u is needed. Using the envelope theorem, differentiation of the uniform-utility condition above shows that $p_u = -1/qv_c < 0$.

Using $p(x, u)$, the developer's profit can be written $p(x, u)h(S) - (1 + \theta)(iS + r)$, so that the first-order condition is $p(x, u)h'(S) = (1 + \theta)i$. This condition determines S as a function of x, u and the property-tax rate θ, written $S(x, u, \theta)$. Differentiation shows that

$$S_x = -\frac{p_x h'}{ph''} < 0, \quad S_u = -\frac{p_u h'}{ph''} < 0, \quad S_\theta = \frac{i}{ph''} < 0, \tag{2}$$

where the signs make use of the results in (1). Thus, improvements per acre decline with x in response to the decline over distance in the price of housing, yielding shorter buildings far from the city center. In addition, by reducing p at all locations, an increase in utility leads to lower improvements throughout the city. Finally, by raising the cost of improvements, an increase in the property-tax rate depresses improvements at all locations, as noted above.

Urban land rent is determined by the developer's zero-profit condition, which is written $r = [p(x, u)/(1+\theta)]h(S(x, u, \theta)) - iS(x, u, \theta)$. Differentiation using the envelope theorem yields

$$r_x = \frac{p_x h}{1+\theta} < 0, \quad r_u = \frac{p_u h}{1+\theta} < 0, \quad r_\theta = -\frac{ph}{(1+\theta)^2} < 0, \tag{3}$$

again using (1). Thus, mirroring the decline of p, land rent falls as distance increases, and an increase in u (by depressing p) reduces r throughout the city. A higher property-tax rate also leads to lower land rent at all locations. Note that since u, and thus p, is held fixed in computing r_θ, the lower land rent in (3) reflects full capitalization of the tax increase. Ultimately, however, u responds to θ, so that full capitalization does not occur.

In standard fashion, the urban equilibrium is determined by two requirements. First, urban land rent must equal the exogenous agricultural rent r^a at the edge of the city, denoted $\bar{x}$. This condition is written

$$r(\bar{x}, u, \theta) - r^a = 0, \tag{4}$$

and it is illustrated in Figure 2, which shows several land-rent curves (the Figure is discussed further below). Note that (4) applies regardless of whether or not the property tax is levied on agricultural land. In either case, equality of net returns to the landowner must hold at $\bar{x}$.

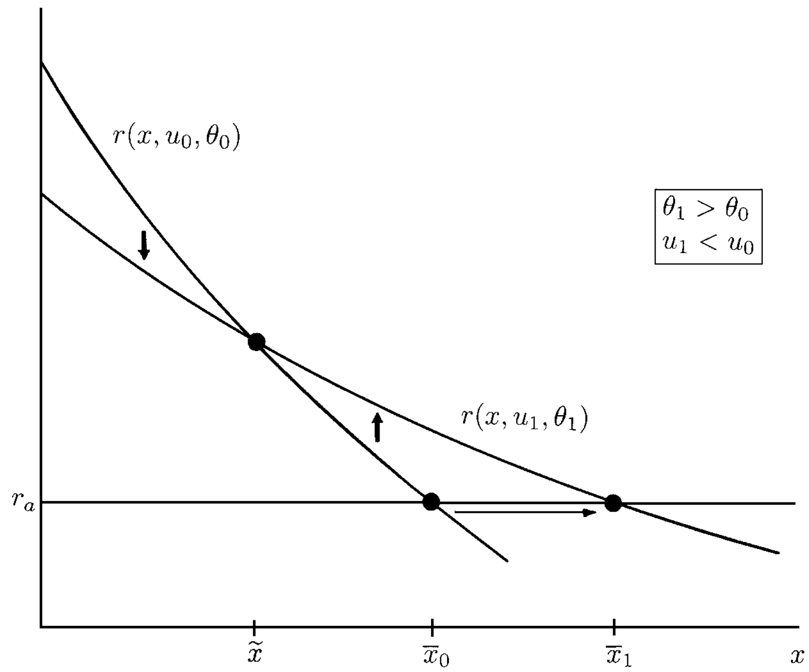

Figure 2. Rotation of the land-rent curve.

The second equilibrium condition requires that the city's exogenous population N fits inside $\bar{x}$. This condition is written

$$\int_0^{\bar{x}} \phi x \frac{h[S(x, u, \theta)]}{q(x, u)} \, dx - N = 0. \tag{5}$$

In (5), ϕ is the constant number of radians of land available for housing at each distance, so that $\phi x(h/q)\,dx$ gives the population occupying the ring of available land at x (recall that h/q equals population density).[4] The integral in (5) thus gives the population that fits inside $\bar{x}$.

The equilibrium conditions (4) and (5) can be rewritten, respectively, as

$$G(\bar{x}, u, \theta) = 0 \tag{6}$$

$$F(\bar{x}, u, \theta) = 0, \tag{7}$$

and they determine $\bar{x}$ and u as functions of the property-tax rate θ. The solution also depends on the levels of N and r^a, but these parameters are held fixed. The effect of the property tax on the spatial size of the city is revealed by the sign of the derivative $\partial\bar{x}/\partial\theta$, which is evaluated in the next section.

Several observations are useful before undertaking this exercise.[5] The first observation concerns the appropriateness of assuming an exogenous city population N. If the goal were to analyze the impact of a higher property tax in a single city, then this closed-city approach would be inappropriate. The reason is that, by disrupting the intercity equilibrium, the unilateral tax increase would generate incentives for migration, altering N. However, if the goal is to analyze the effect of universal use of the property tax, then the closed-city model is appropriate. In this case, a simultaneous increase in the common property-tax rate across the economy's fixed number of identical cities would generate no incentives for migration, leaving each city's population unaffected. For this reason, the urban-sprawl impact of economy-wide reliance on the property tax is best analyzed using a closed-city approach.[6]

The second observation is that the model assumes absentee landownership, as is common in the literature. Therefore, the rent generated by the land does not appear as income for the urban residents, accruing instead to individuals living elsewhere. An important task for future research would be to relax this assumption by analyzing a "fully closed" city, where residents earn rental income (see Pines and Sadka, 1986).[7]

The final observation concerns disposition of the revenue raised by the property tax. To derive the main results, it is assumed that tax revenue disappears, not being returned to the urban residents in the form of public expenditure. Once the analysis is complete, however, the conclusions can be used to predict the effect of property-tax changes in a model where this tax-revenue leakage is eliminated. In particular, with the city engaging in a fixed level of public spending, the analysis can be used to predict the effect of switching from the property tax to a nondistortionary land tax as a revenue source. Under such a switch, the property-tax rate would be reduced to zero, and a land tax would be introduced and its rate raised enough to maintain the level of public spending. However, since the land tax has no effect on the city's spatial size, the net impact of this regime switch corresponds to the effect of eliminating the property tax. Thus, the sign of $\partial\bar{x}/\partial\theta$, derived under the assumption that tax revenue disappears, tells whether cities shrink or grow under an efficiency-enhancing, revenue-neutral shift to a land tax. This point is discussed in more detail below.

3. Comparative-Static Analysis

Differentiating (6) yields

$$G_{\bar{x}} = \bar{r}_x < 0, \quad G_u = \bar{r}_u < 0, \quad G_\theta = \bar{r}_\theta < 0, \tag{8}$$

where the bar indicates that the land-rent derivatives are evaluated at $x = \bar{x}$ (the signs rely on (3)). In addition, differentiation of (7) yields

$$F_{\bar{x}} = \frac{\phi \bar{x} \bar{h}}{\bar{q}} > 0, \quad F_u = \int_0^{\bar{x}} \phi x \frac{h' S_u q - h q_u}{q^2}\, dx < 0, \quad F_\theta = \int_0^{\bar{x}} \phi x \frac{h' S_\theta}{q}\, dx < 0, \tag{9}$$

where $\bar{h}$ and $\bar{q}$ give h and q evaluated at $x = \bar{x}$ (note that $\bar{h} = h[S(\bar{x}, u, \theta)]$). The signs in (9) follow from (2).

Application of Cramer's rule to (6) and (7) yields

$$\frac{\partial u}{\partial \theta} = \frac{-F_{\bar{x}} G_\theta + G_{\bar{x}} F_\theta}{A} < 0, \tag{10}$$

where $A = F_{\bar{x}} G_u - G_{\bar{x}} F_u < 0$. The negative sign of A follows from (8) and (9), which also yield a positive sign for the numerator of (10). Thus, an increase in the property-tax rate reduces the utility level in the city, an effect that reflects both its distortionary nature and the revenue leakage it creates.

Similarly, the effect of θ on $\bar{x}$ is given by

$$\frac{\partial \bar{x}}{\partial \theta} = \frac{-F_\theta G_u + G_\theta F_u}{A}. \tag{11}$$

Given (8) and (9), it is easily seen that the numerator expression in (11) is ambiguous in sign. Evaluating the negative of this expression, which gives the sign of (11) in view of $A < 0$, yields the following conclusion:

$$\begin{aligned} \frac{\partial \bar{x}}{\partial \theta} &\simeq \int_0^{\bar{x}} \frac{\phi x h'}{q} (S_\theta \bar{r}_u - S_u \bar{r}_\theta)\, dx + \bar{r}_\theta \int_0^{\bar{x}} \frac{\phi x h q_u}{q^2}\, dx \\ &= \int_0^{\bar{x}} \frac{\phi x h' i \bar{h}}{(1+\theta) p q h''} \bar{p}_u p_u \left[\frac{p}{p_u} - \frac{\bar{p}}{\bar{p}_u} \right] dx + \bar{r}_\theta \int_0^{\bar{x}} \frac{\phi x h q_u}{q^2}\, dx, \end{aligned} \tag{12}$$

where the second line makes use of (2) and (3) (here, the symbol $\simeq$ means 'has same sign as'). Note that, except for cases where a bar is present, variables inside the integrals in (12) have an implicit x argument, thus depending on location. Since $\bar{r}_\theta$ is negative and q_u positive, the second half of (12) is negative. In addition, since p_u, $\bar{p}_u$, and h'' are all negative, the sign of the expression multiplying the bracketed term in the first integral is negative. However, the bracketed term itself, which equals the difference between p/p_u at a given $x \le \bar{x}$ and its value at $x = \bar{x}$, is generally ambiguous in sign. As a result, (12) cannot be signed in general.

Despite this sign ambiguity, results are available for special cases, as shown in the ensuing analysis. The first two cases assume that the key decision variables, dwelling size q and structural density S, are alternately fixed in exogenous fashion. The results for these cases confirm the simple intuition discussed in the introduction. Then, the analysis turns to cases where both q and S are flexible, with their variability governed by substitution possibilities in consumption and production. As will be seen, the analysis finds a relatively clear link between the sign of $\partial\bar{x}/\partial\theta$ and the elasticity of substitution in consumption, which determines the magnitude of the dwelling-size effect. However, with exception of the polar case where capital and land must be used in fixed proportions, a simple statement about the effect of input substitutability in production is not available.

3.1. The Fixed-q Case

Suppose that dwelling sizes in the city are fixed and constant across locations, with $q = q^*$ at each x. Uniform utility then requires a constant c, denoted c^*, which is achieved by appropriate spatial variation in p. Using the above formula for p_u, it follows that $p_u = -1/q^* v_c(c^*, q^*) = \text{constant}$. But with p_u a negative constant, the bracketed term in the first integral in (12) is proportional to $\bar{p} - p$, a negative expression given $p_x < 0$. The first integral is then positive, and since the fixed q implies $q_u = 0$, the second integral in (12) is zero. The expression in (12) is then positive, yielding

Proposition 1 *If dwelling size is fixed and constant across locations, then* $\partial\bar{x}/\partial\theta > 0$*. The city therefore expands spatially in response to an increase in the property-tax rate.*

This result confirms the intuition suggested by the nonspatial model, namely, that by depressing the level of improvements, the property tax reduces the city's average population density and thus causes it to expand spatially. However, the spatial pattern of the impact on improvements turns out to be complex, showing that the simple intuition masks some underlying subtleties.

To understand this point, note from (2) that, while the direct effect of the higher θ is to reduce S ($S_\theta < 0$), the reduction in utility from (10) has the opposite impact given $S_u < 0$. This second effect, which was absent in the previous partial-equilibrium analysis, arises because p is now endogenous, rising in response to the decline in u and pushing up the desired level of improvements. The net impact of these two forces has a particular spatial pattern, which can be deduced by analyzing the pattern of changes in land rent (this conclusion follows because r and S move in the same direction as θ changes). From above, land rent at a particular $x = \hat{x}$ is given by $\hat{r} = \hat{p}\hat{h}/(1+\theta) - i\hat{S}$, and its total derivative is

$$\frac{d\hat{r}}{d\theta} = \left(\frac{\hat{p}_u}{1+\theta}\frac{\partial u}{\partial\theta} - \frac{\hat{p}}{(1+\theta)^2}\right)\hat{h}, \tag{13}$$

where $\hat{p}_u$ is p_u evaluated at $x = \hat{x}$ (the envelope theorem is used). Since S moves in step with changes in land rent, the sign of (13) also gives the sign of $d\hat{S}/d\theta$. After substituting the utility derivative from (10) and suppressing the $\hat{h}$ factor, considerable manipulation of

(13) shows that

$$\frac{d\hat{r}}{d\theta} \simeq \frac{\phi\bar{x}\bar{h}^2}{\bar{q}(1+\theta)^2}\hat{p}_u\bar{p}_u\left[\frac{\hat{p}}{\hat{p}_u}-\frac{\bar{p}}{\bar{p}_u}\right]+\int_0^{\bar{x}}\frac{\phi x h' i \bar{r}_x}{p^2 q h''}\hat{p}_u p_u\left[\frac{\hat{p}}{\hat{p}_u}-\frac{p}{p_u}\right]dx$$
$$+\bar{r}_x\hat{p}\int_0^{\bar{x}}\frac{\phi x h q_u}{q^2}\,dx. \tag{14}$$

Using (14), it is easily seen that the land rent curve rotates in a counterclockwise direction when θ increases, as shown in Figure 2. To see this result, observe first that when q is fixed, the third term in (14) equals zero. Next, let the remaining terms be evaluated at the CBD, where $x = 0$. Since p_u is negative and constant when q is fixed, the first term has the sign of $\bar{p} - p^{cbd}$, which is negative given $p_x < 0$ (p^{cbd} gives the housing price at the CBD). Similarly, the integrand in the second term has the sign of $p - p^{cbd}$, which is negative over the range of integration. Thus, (14) is negative at $x = 0$, indicating that land rent falls at the CBD. Conversely, at $x = \bar{x}$, the first term in (14) equals zero, while the integrand has the sign of $p - \bar{p} > 0$. As a result, (14) is positive at $\bar{x}$, indicating that land rent increases at the edge of the city. Since (14) is a monotonically increasing function of $\hat{x}$, it follows that land rent falls inside of some $\tilde{x} < \bar{x}$, while rising between $\tilde{x}$ and $\bar{x}$, yielding the rotation shown in Figure 2.[8] As a result of this rotation, the curve's intersection with the agricultural rent line moves outward, so that $\bar{x}$ increases.

While this outcome confirms Proposition 1, the Figure also indicates that the impact of the higher property tax on improvements has a complex spatial pattern. Recalling that S moves in step with land rent, it follows that improvements fall inside of $\tilde{x}$ when θ increases, as expected, but that S *rises* outside of $\tilde{x}$. Therefore, the increase in the property-tax rate does not reduce the level of improvements everywhere in the city. However, the downward pressure on population density resulting from the decline in S in the central part of the city is sufficient to generate an increase in $\bar{x}$.

3.2. The Fixed-S Case

Now consider the alternate case where q is flexible but improvements per acre are fixed at each location. In this case, the S derivatives in the first line of (12) are zero, so that (12) reduces to the second term, which is negative. This fact yields

Proposition 2 *If improvements per acre are fixed at each location, then $\partial\bar{x}/\partial\theta < 0$. The city therefore shrinks spatially in response to an increase in the property-tax rate.*

Note that in contrast to Proposition 1, which required a locationally invariant q, this result does not require that S is constant across locations.

The outcome in Proposition 2 is easy to understand intuitively given the decline in utility from (10), which still occurs when S is fixed. With utility falling, q decreases at each location given $q_u > 0$, and this raises population density everywhere in the city. Higher densities in turn allow a reduction in $\bar{x}$. Stated differently, the dwelling-size reduction that generates higher density is a consequence of forward shifting of the property tax. When θ increases,

p rises everywhere in the city given that u falls and $p_u < 0$, and this price increase leads to a reduction in dwelling sizes.

Referring to (14), land rent falls at the edge of the city in the fixed-S case, which provides another way of understanding the need for a reduction in $\bar{x}$. To see this decline, observe that the second term in (14) is not present when S is fixed. Since the first term equals zero when $\hat{x} = \bar{x}$, (14) reduces to the negative third term when evaluated at $\bar{x}$, indicating that land rent falls at the city's edge. However, at locations inside $\bar{x}$, the sign of the first term in (14) is ambiguous, so that the direction of land-rent changes in the interior of the city cannot be signed.

3.3. *The Case of CES Preferences*

While the fixed-q case eliminates the dwelling-size effect, leading to an unambiguous increase in $\bar{x}$ when θ rises, the magnitude of dwelling-size effect is an important determinant of the sign of $\partial\bar{x}/\partial\theta$ when q is flexible. This magnitude depends in part of the degree of substitutability between q and c in consumption, which helps determine how much q declines in response to the tax-induced increase in p. To explore the effect of substitutability, suppose that preferences are CES, so that $v(c, q) \equiv [\alpha c^{-\beta} + (1-\alpha)q^{-\beta}]^{-1/\beta}$, with $\sigma = 1/(1+\beta)$ giving the elasticity of substitution. Recall that β satisfies $-1 \leq \beta < \infty$, implying $0 \leq \sigma < \infty$. Then, the following result can be established:

Lemma *If preferences are CES, then the ratio p/p_u is increasing in x when $\sigma < 1$. The ratio is decreasing (constant) in x when $\sigma > (=)1$.*

This result is proved by using the first-order condition $v_q/v_c = p$ and the formula for p_u to rewrite p/p_u as $-qv_q$. In the CES case, $-qv_q$ is proportional to $-u^{1+\beta}q^{-\beta} = -u^{1/\sigma}q^{(\sigma-1)/\sigma}$. Given the constancy of u and $q_x > 0$, the lemma follows.[9]

Thus, the spatial behavior of the ratio p/p_u, which enters the ambiguous term in (12), depends on the elasticity of substitution between housing and the nonhousing commodity in the CES case. Applying the lemma then yields

Proposition 3 *If $\sigma \geq 1$, then $\partial\bar{x}/\partial\theta < 0$. If $\sigma < 1$, then $\partial\bar{x}/\partial\theta$ remains ambiguous in sign.*

A higher property tax thus causes the city to shrink spatially when c and q are highly substitutable, with the outcome ambiguous otherwise. This result is established by noting that, since p/p_u is constant or decreasing in x when $\sigma \geq 1$, the bracketed term in the first integral of (12) is positive or zero, making the integral nonpositive. Since the second term in (12) is negative, it follows that $\partial\bar{x}/\partial\theta < 0$. Conversely, when $\sigma < 1$, the first integral in (12) is positive, making the entire expression ambiguous in sign. Note that the $\sigma \geq 1$ case, where $\bar{x}$ falls with θ, includes the special case of Cobb-Douglas preferences, which correspond to $\sigma = 1$. This conclusion helps explain the simulation results of Arnott and MacKinnon (1977), who assumed Cobb-Douglas preferences and found that their simulated city shrank spatially when θ increased.

The intuition underlying Proposition 3 is easily stated. Utility falls when θ increases, and this movement to a lower indifference tends to reduce q, holding housing prices constant. But p rises at all locations, as noted above, and this price increase generates substitution, moving the consumption bundle along the lower indifference curve in the direction of a smaller q. These effects together tend to reduce q, raising population density and putting downward pressure on $\bar{x}$. But their combined effect is strongest when the elasticity of substitution is large, in which case the second effect is substantial. In this situation, downward pressure on $\bar{x}$ from smaller dwellings has the best chance of dominating the upward pressure from lower improvements. Proposition 3 shows that this dominance is unambiguous when $\sigma \geq 1$.

To relate the above results to Figure 2, note that when $\sigma \geq 1$, land rent falls at the edge of the city, accounting for the decline in $\bar{x}$. This follows because the second integral in (14) is negative or zero at $\hat{x} = \bar{x}$ given the lemma, with the first and third terms zero and negative respectively. While land rent also declines at all interior locations with Cobb-Douglas preferences, interior rent changes are ambiguous when $\sigma > 1$.[10]

It is interesting to note that, when both S and q are endogenous, it is incorrect to associate the first and second terms in (12) with their separate positive and negative effects on $\bar{x}$. This follows because the first term in (12), which represented the S effect in the analysis leading to Proposition 1, is negative or zero when $\sigma \geq 1$, rather than positive. Thus, a subtle interaction between the improvements and dwelling-size effects arises when both variables are endogenous.

3.4. The Role of Input Substitutability in Housing Production

A remarkable aspect of Proposition 3 is that an unambiguous conclusion emerges for the $\sigma \geq 1$ case regardless of the magnitude of the improvements effect, as determined by the extent of input substitutability in housing production. To gain some understanding of the role of input substitutability, it should be noted that Proposition 2 already gives results for a polar case. In particular, while the Proposition requires that S is fixed at each location, the subcase where S is *fixed and constant* throughout the city corresponds to a fixed-proportions housing technology, with right-angled isoquants. In this Leontief case, the improvements effect is absent, and Proposition 2 shows that the city shrinks in response to an increase in θ.

It is not possible, however, to go beyond this zero-substitution case to show analytically the effect of greater input substitutability. To see the obstacle, suppose that a CES production function were assumed, generating an intensive function h that depends on the substitution elasticity ρ. Substituting in (12), the RHS expressions would then depend on ρ. However, since ρ affects the equilibrium value of u as well as helping to determine S, the connection between ρ's magnitude and the sign of (12) is not straightforward. As a result, a simple statement relating the sign of $\partial\bar{x}/\partial\theta$ to the extent of input substitutability is not available.

4. Numerical Examples

Refocusing on the case of CES preferences, where determinate results are available, recall that Proposition 3 provides concrete conditions under which a higher property tax causes spatial shrinkage of the city when both q and S are flexible. However, the results leave open the question of whether an increase in θ can ever generate the reverse outcome, leading to urban spatial expansion. As seen above, if this outcome is to occur, it must happen when substitutability in consumption is low and the dwelling-size effect is muted.

To explore this question, this section presents numerical examples based on Leontief preferences, which correspond to the limiting CES case where $\sigma = 0$. Since substitution is absent in this case, the price-induced reduction in q discussed above is not present. This absence limits the magnitude of the dwelling-size effect and provides the best opportunity for spatial expansion of the city in response to a tax increase.[11]

With Leontief preferences, $v(c, q) \equiv \min\{c, \delta q\}$. Given the absence of substitution, it follows that the urban consumption bundle is the same at all locations, with the bundle lying at an indifference-curve corner. As a result, the relationship $u = c = \delta q$ holds, allowing the budget constraint to be written $u + pu/\delta = y - tx$. Rearranging, it follows that $p(x, u) \equiv \delta[(y - tx)/u - 1]$. Using this price function, solutions for S and r are computed using a Cobb-Douglas housing production function, whose intensive form is $h(S) = gS^{\gamma}$, where $0 < \gamma < 1$.

For the base-case example, the city population N is set equal to 1,000,000, and household income y is set at \$45,000 per year. The commuting cost parameter t equals \$500 per mile per year, a value that includes both money and time cost components.[12] Agricultural rent equals \$150 per acre per year, corresponding to a land value of \$3000 per acre under a 5 percent discount rate (this yields a rent per square mile of \$96,000). The Leontief parameter δ is set at 5 to assign a relatively large preference weight to nonhousing consumption. The production-function exponent γ is set at 0.5, and the parameters g and i are set arbitrarily to generate realistic cities. Their values are 25,000 and 1, respectively. Finally, ϕ is set at 2π, reflecting a circular city.

Solutions to the equilibrium conditions (4) and (5) are computed for three values of the property-tax rate θ: 0, 0.3, and 0.5. Since the property tax in the model is levied on rents, the tax rate as a fraction of value is found by multiplying these figures by the discount rate, assumed to be 5 percent. The θ values of 0.3 and 0.5 then yield realistic property-tax rates of 1.5 and 2.5 percent.

The first panel of Table 1 shows the base-case solutions. With a zero tax rate, the city has a radius of 7.08 miles. When the tax rate rises to 0.3, the city expands spatially, with $\bar{x}$ growing to 7.61 miles, an increase of 7.5 percent. When the tax rate is raised further to 0.5, $\bar{x}$ grows to 7.91, an additional increase of 4 percent. Therefore, a higher property tax leads to spatial expansion of the city.

To understand these outcomes, note first that the reduction in dwelling size in response to the increase in θ is very slight in the Leontief case, as seen in Table 1. Note that since q and u are proportional, these q changes imply a slight utility decline as θ rises. Although q is mostly unaffected, improvements exhibit the behavior already seen in the fixed-q case as θ increases. In particular, S falls in the central part of the city, rising near the city's edge. This pattern can be seen in Table 2, which shows population density $(h(S)/q)$ at a number of locations under the different tax rates (the values give population per square mile). With the uniform q falling only slightly, the illustrated changes in density closely track the changes in S. The Table shows that, as θ increases from zero to 0.3, density falls at x values of 0, 2, and 4 miles, while rising at $x = 6$ and $x = 7$, near the edge of the original city. As θ rises

Table 1. Numerical examples: Effects on $\bar{x}$ and q.†

	Base Case		$\delta = 2$		$N = 1{,}500{,}000$	
	$\bar{x}$	q	$\bar{x}$	q	$\bar{x}$	q
$\theta = 0.0$	7.08	8.24	12.22	19.20	8.09	8.16
$\theta = 0.3$	7.61	8.18	13.05	18.95	8.70	8.08
$\theta = 0.5$	7.91	8.14	13.51	18.75	9.05	8.04
	$r^a = 250$		$t = 700$		$y = 55{,}000$	
	$\bar{x}$	q	$\bar{x}$	q	$\bar{x}$	q
$\theta = 0.0$	6.97	8.24	6.30	8.08	8.09	10.14
$\theta = 0.3$	7.48	8.18	6.78	8.00	8.70	10.06
$\theta = 0.5$	7.76	8.14	7.05	7.96	9.04	10.02

Other base-case parameter values

N	y	r^a	t	i	δ	γ	g
1,000,000	45,000	150	500	1	5	0.5	25,000

†q is the actual value times 10^{-3}.

Table 2. Numerical examples: Effects on population density in base case.†

	Population Density		
	$\theta = 0.0$	$\theta = 0.3$	$\theta = 0.5$
$x = 0$	17183	14596	13360
$x = 2$	12593	11008	10223
$x = 4$	8003	7421	7085
$x = 6$	3413	3834	3947
$x = 7$	1118	2041	2378
$x = 7.5$	—	1144	1594

†Density is measured in people per square mile.

to 0.5, the density contour rotates further. Thus, the outcome recapitulates the pattern seen in Figure 2.

These density changes, which operate mainly through changes in improvements, reflect a rotation of the land-rent curve, as can be seen from (14). As in the fixed-q case, the first two terms generate a land-rent rotation under the present numerical specification, with the third term generating a downward shift. Because the change in q is small, the rotation effect dominates the downward shift, so that the net effect is for the land-rent curve to rotate. This in turn yields a rotation of both the S and density contours.[13]

Returning to Table 1, the additional panels provide sensitivity analysis, showing the effects of altering the base-case parameter values. Increasing the weight of housing in preferences by setting $\delta = 2$ raises q, increasing the initial land area of the city. However, as can be seen from the second panel of Table 1, a higher property tax continues to raise $\bar{x}$. Similarly, raising the city's population to 1.5 million makes it spatially larger and lowers q (and hence u), but a higher θ continues to generate urban expansion. A higher agricultural rent r^a makes the city more compact and negligibly reduces q and u, but $\bar{x}$ still increases as the property-tax rate rises. The city takes up less space, and q and u are lower, with a higher commuting-cost parameter t, but the city again expands when θ is raised. An increase in income y raises the city's spatial size along with q and u, but a higher property tax still leads to urban expansion.

Thus, under realistic parameter values, including many other combinations that are not reported, an increase in the property-tax rate leads to spatial expansion of the city.[14] While the numerical examples establish this conclusion for the polar Leontief case, a continuity argument implies that the same outcome will also emerge in intermediate cases where substitutability in preferences is low but not totally absent.[15] Since empirical evidence points toward a low σ, the numerical results thus suggest that property taxation may encourage urban sprawl.[16]

5. The Effects of a Revenue-Neutral Switch to a Land Tax

As explained above, the discussion implicitly assumes that the revenue raised by the property tax disappears, with the revenue leakage rising as θ increases. However, to draw realistic conclusions, both positive and normative, regarding the property tax and urban sprawl, this revenue leakage must be eliminated. Ideally, such conclusions should be based on a revenue-neutral comparison between the property tax and an alternate, nondistortionary tax regime. To this end, suppose that the city collects a fixed amount of tax revenue to support an exogenous level of public spending. This revenue is initially raised by the property tax, but the city then switches to an equal-yield, nondistortionary land tax.

To appraise the effects of this switch, the first step is to note that, since the land tax is fully capitalized in land rents, the urban equilibrium is unaffected by the magnitude of the land-tax rate. In particular, it can be shown that the land-tax rate τ has no effect on $\bar{x}$ and u.[17] Then, assuming that $\partial\bar{x}/\partial\theta > 0$ holds under the property-tax regime, as in the numerical examples, substitution of the land tax in place of the property tax leads to a spatial shrinkage of the city. To see this result, observe that eliminating the property tax by setting the rate at zero would reduce $\bar{x}$, while imposing a land tax and raising the rate τ sufficiently to recover the lost property-tax revenue would have no effect on the city's spatial size. Thus, a revenue-neutral switch between the two tax regimes would shrink the city.

Since utility rises as the property-tax rate is lowered and is unaffected as the land-tax rate increases, consumers benefit from this change in tax regimes. It can be shown that this positive welfare verdict applies more generally in a setting where the well-being of both urban residents and absentee landowners is taken into account, with the land tax yielding a Pareto-optimal outcome. While affirming the well-known virtues of the land tax, this conclusion also establishes that, in the case where $\partial\bar{x}/\partial\theta > 0$, the urban expansion caused by the property tax is socially undesirable. In this case, property taxation would belong on the list of factors causing inefficient spatial expansion of cities. Of course, the reverse case may hold, with the property tax reducing rather than increasing the city's spatial size. In this situation, the tax causes an inefficient shrinkage of the city. Note that the latter statement presumes the absence of other distortions such as unpriced road congestion that make cities too large (in which case shrinkage would be welcome).

6. Conclusion

This paper has explored the connection between the property tax and urban sprawl. While the tax's depressing effect on improvements reduces population density, spurring the spatial expansion of cities, a countervailing effect from lower dwelling sizes may dominate, raising densities and making cities smaller. The analysis shows that this latter outcome is guaranteed under CES preferences when the elasticity of substitution σ is high. But numerical examples based on Leontief preferences (where σ is zero) suggest that the property tax may encourage urban sprawl in the case where substitution between housing and other goods is low. Thus, the distortions generated by the property tax may include inefficient spatial expansion of cities, suggesting the tax may belong on the list of causal factors identified by critics of urban sprawl.

Empirical work based on the current analysis would be a useful avenue for future research. Using a cross-section of urbanized areas, an empirical study could estimate a regression equation relating a city's spatial size to a property-tax measure and other relevant variables, such as population, income, agricultural land rent, and a commuting-cost proxy.[18] The fact that effective property-tax rates are unavailable for a national sample of cities poses an obstacle to such a study. However, with suitable precautions, a variable measuring the average effective tax rate within each county could be constructed from available data. Assuming that this average rate is relevant for the county's urbanized areas, the above regression could be run. A nonzero coefficient for the property-tax rate would validate the usefulness of the present analysis, with a positive estimate indicating that the property tax encourages urban sprawl.

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Notes

1. For an elaboration of these points, as well references to the literature, see Brueckner (2000, 2001b). Other indicators of sprawl, such as a redistribution of population toward the suburbs without an increase in the city's overall spatial size, may also be useful, but the present analysis focuses solely on spatial expansion.
2. See Wheaton (1998) and the earlier literature he cites for a formal demonstration of this point.
3. See, for example, Brueckner (1986), Case and Grant (1991), Oates and Schwab (1997), Mills (1998), and Nechyba (1998). Another literature explores the effect of land taxation in a dynamic setting, where the tax does affect resource allocation by altering the timing of land development. For an analysis of this effect, see Bentick (1979), Mills (1981), Wildasin (1982), and Anderson (1986).
4. A circular city has $\phi = 2\pi$.

5. Each of the following points also applies to the model of Arnott and MacKinnon (1977), whose assumptions parallel the present ones.
6. For this view of the analysis to make sense, an additional assumption is needed. In particular, housing capital must be supplied in perfectly elastic fashion from outside the economy, justifying the implicit assumption that its cost per unit i is fixed. If the economy instead faced a fixed supply of capital, then the gross-of-tax price $(1+\theta)i$ would also be fixed, implying that an increase in θ would be fully shifted backward onto owners of capital. In this case, the property tax would function as a pure land tax, having no effect on the spatial size of the city (see below).
7. Note that the income from housing capital also accrues to absentee owners. In a completely closed model, urban residents would earn both capital income and income from land rents.
8. To see this fact, note that with p_u constant, $\hat{x}$ enters (14) only as an argument of $\hat{p}$ in the first and second terms. Since $\hat{p}$ is multiplied by negative quantities, and since $p_x < 0$, it follows that (14) is increasing in $\hat{x}$.
9. We thank David Pines for this abbreviated proof, which replaced the longer original one.
10. This conclusion follows because the first two terms in (14) are zero for all values of $\hat{x}$ in the Cobb-Douglas case (a consequence of p/p_u being constant when $\sigma = 1$). When $\sigma > 1$, however, the first two terms in (14) are positive at $\hat{x} = 0$, so that the land-rent change is ambiguous at the CBD, and consequently at other interior x values.
11. Note while q remains flexible in the zero-substitution case, the ratio of q to c is fixed, mirroring the fixity of S (the capital-to-land ratio) in the fixed-proportions production case.
12. Assuming an out-of-pocket cost of \$0.30 per mile and 250 round trips per year leads to value of \$150 per year for the money cost of commuting per mile. A yearly income of \$45,000 implies an hourly wage of \$22.50, which yields a time cost per mile of \$0.75 assuming a traffic speed of 30 miles per hour. Time cost is then \$375 per mile per year, yielding a total money plus time cost of \$525, which is rounded down to \$500.
13. Recall from above that rotation of the land rent curve arising from the first two terms in (14) cannot be established in general when q is endogenous.
14. In unrealistic numerical examples, the opposite outcome can occur. For example, if g is set equal to 1 instead of 25,000, the original city (with $\theta = 0.0$) has an unrealistically large radius of 73.54 miles. When θ is raised to 0.5, $\bar{x}$ shrinks slightly to 70.28 miles. In addition, it should be noted that, despite the realistic city sizes in the present simulation results, other aspects of the simulated cities are implausible. For example, housing expenditure represents a small fraction of income, only 8 percent at the CBD in the base case, and less than 1 percent at $\bar{x}$.
15. Simulating such cases is difficult with CES preferences, however, because closed-form solutions for the variables of interest are not available. Such a simulation exercise, which is beyond the scope of the present paper, could be a subject for future work.
16. The value of σ can be inferred from evidence on housing demand. First, the Slutsky equation yields $\eta_p = \tilde{\eta}_p - \mu\eta_y$, where η_p and $\tilde{\eta}_p$ are the uncompensated and compensated price elasticities of housing demand, η_y is the income elasticity, and μ is housing's budget share. In addition, the elasticity of substitution satisfies $\sigma = -\tilde{\eta}_p/(1-\mu)$. Eliminating $\tilde{\eta}_p$ using the first equation, σ can be written as $-(\eta_p + \mu\eta_y)/(1-\mu)$. Then, substituting representative values of 0.5 and −0.5 for η_y and η_p (see Mayo (1981)), the implied σ value is 0.5 regardless of the magnitude of μ.
17. To see this result, focus for simplicity on the case where agricultural rent r^a equals zero. Then, note that with a land tax, the first-order condition for choice of S is given by $p(x,u)h'(S) = i$ (see the discussion in Section 2.1). Structural density is then independent of τ, being given by $S(x,u)$, and land rent equals $r(x,u,\tau) \equiv [p(x,u)h(S(x,u)) - iS(x,u)]/(1+\tau)$. Since this equation yields $r_\tau = -r/(1+\tau)$, it follows that $r_\tau = 0$ holds when $r = 0$. Given that the urban equilibrium conditions now consist of $r(\bar{x},u,\tau) = 0$ and (5) with the tax rate suppressed, it follows that differentiation yields $\partial u/\partial\tau = \partial\bar{x}/\partial\tau = 0$.
18. Brueckner and Fansler (1983), in an earlier study of the determinants of urban spatial sizes, estimated such a regression without including a property-tax variable.

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