EC2410-Spring 2023
Problem Set 3

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

Answers which do not achieve these goals will not be awarded full credit.

Problems

1. In this problem, we will work through an example of the discrete choice model with heterogeneous agents. Consider a discrete linear city with three neighborhoods $i \in \{1, 2, 3\}$. Let x_i denote a neighborhood's distance from the CBD, with x_1 = 1, x_2 = 2, x_3 = 3. The cost to commute one unit distance is τ. The city is populated by households indexed by j. Each household chooses a neighborhood i, pays land rent R_i, and commutes to the center, at location 0, to earn wage w. A household's utility is $V_{ij} = A_i \cdot c_i z_{ij}$ where $A_i = i$ is the amenity value in location i, c_i is consumption and z_{ij} is the household and location specific valuation. All z_{ij} are drawn from a Frechet distribution, $F(z) = e^{-Tz^{-\epsilon}}$.

   (a) Let consumption be $c_i = w - R_i + i\tau$. Set up the household's problem.

   $$V_{ij} = A_i.z_{ij} \cdot c_i$$

   Using $c_i = w - R_i + i\tau$ and $A_i = i$,

   $$V_{ij} = i \cdot z_{ij} \cdot (w - R_i - i\tau)$$

   Therefore, the household's problem is to choose a discrete location i that maximizes its utility.

   $$\max \{V_{1j}, V_{2j}, V_{3j}\}$$

   (b) Using the big theorem from the lecture, solve for the share of household s_i in each location.

   $$s_i = \frac{[i \cdot (w - R_i - i\tau)]^\epsilon}{\sum_{k=1}^{3}[k \cdot (w - R_k - k\tau)]^\epsilon}$$

   (c) Let the share of households in each location $s_1 = s_2 = s_3 = \frac{1}{3}$, wage $w = 5$ and the price of agricultural land $\bar{R} = 1$. Assume that the land rent at $x = 3$ is equal to $\bar{R}$. Solve for R_1, R_2 and R_3 in terms of τ.

   Setting $s_1 = s_2 = s_3 = \frac{1}{3}$ we get the following three equations:

$$s_1 = s_2 \implies w - R_1 - \tau = 2(w - R_2 - 2\tau)$$
$$2R_2 - R_1 = w - 3\tau$$

$$s_2 = s_3 \implies 2(w - R_2 - 2\tau) = 3(w - R_3 - 3\tau)$$
$$2R_2 - 3R_3 = 5\tau - w$$

$$s_1 = s_3 \implies w - R_1 - \tau = 3(w - R_3 - 3\tau)$$
$$3R_3 - R_1 = 2w - 8\tau$$

Plugging in $w = 5$ and $R_3 = \bar{R} = 1$, we get:

$$R_1 = 8\tau - 7$$
$$R_2 = 2.5\tau - 1$$
$$R_3 = 1$$

(d) Solve for consumption in terms of τ.

Substituting R_i from part c and w in $c_i = w - R_i + i\tau$

$$c_1 = 12 - 9\tau$$
$$c_2 = 6 - 4.5\tau$$
$$c_3 = 4 - 3\tau$$

2. The discrete choice problem with Frechet taste heterogeneity requires that

$$s_{ij} = \frac{\left(\frac{A_{ij}\, w}{\tau_{ij} R_j^{\beta}}\right)^{\theta}}{\sum_{m=1}^{3} \sum_{n=1}^{3} \left(\frac{A_{mn}\, w}{\tau_{mn} R_n^{\beta}}\right)^{\theta}}.$$

Using $w = 10$ and $R_j = 1$ for all j, this simplifies to

$$s_{ij} = \frac{\left(\frac{A_{ij}}{\tau_{ij}}\right)^{\theta}}{\sum_{m=1}^{3} \sum_{n=1}^{3} \left(\frac{A_{mn}}{\tau_{mn}}\right)^{\theta}}, \qquad \theta = 8.$$

Since $s_{ij} = \frac{1}{9}$ for all i,j, every numerator must be identical. Then, using Using $A_{11} = \tau_{11} = 1$, we have

$$\left(\frac{A_{ij}}{\tau_{ij}}\right)^{8} = 1 \text{ all } (i,j).$$

Therefore,

$$A_{ij} = \tau_{ij} \quad \text{all } i,j,$$

or equivalently,

$$A = \begin{bmatrix} 1.00 & 1.10 & 1.21 \\ 1.10 & 1.00 & 1.10 \\ 1.21 & 1.10 & 1.00 \end{bmatrix}.$$

To evaluate counterfactual shares under τ', let

$$\tau' = \begin{bmatrix} 1.00 & 1.20 & 1.44 \\ 1.20 & 1.00 & 1.20 \\ 1.44 & 1.20 & 1.00 \end{bmatrix}.$$

Holding A_{ij} fixed at the values above, the counterfactual shares are

$$s'_{ij} = \frac{\left(\frac{A_{ij}}{\tau'_{ij}}\right)^8}{\sum_{m=1}^{3}\sum_{n=1}^{3}\left(\frac{A_{mn}}{\tau'_{mn}}\right)^8} = \frac{\left(\frac{\tau_{ij}}{\tau'_{ij}}\right)^8}{\sum_{m=1}^{3}\sum_{n=1}^{3}\left(\frac{\tau_{mn}}{\tau'_{mn}}\right)^8}.$$

Numerically,

$$s' \approx \begin{bmatrix} 0.182110 & 0.090787 & 0.045260 \\ 0.090787 & 0.182110 & 0.090787 \\ 0.045260 & 0.090787 & 0.182110 \end{bmatrix}.$$

Thus, the increase in transportation costs increases the share of people who work where they live, just as it should.