EC2410-Spring 2023
Problem Set 3

(Updated 27 February 2026)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

Answers which do not achieve these goals will not be awarded full credit.

Problems

1. Consider a discrete linear city with three neighborhoods $i \in \{1, 2, 3\}$. Let x_i denote a neighborhood's distance from the CBD, with x_1 = 1, x_2 = 2, x_3 = 3. The cost to commute one unit distance is τ. The city is populated by households indexed by ν. Each household chooses a neighborhood i, pays land rent R_i, and commutes to the center, at location 0, to earn wage w. A household's utility is $V_i(\nu) = A_i \varepsilon_i(\nu) c_i$ where $A_i = i$ is the amenity value in location i, c_i is consumption and $\varepsilon_i(\nu)$ is the household and location specific valuation. All $\varepsilon_i(\nu)$ are drawn from a Frechet distribution, $F(\varepsilon) = e^{-T\varepsilon^{-\theta}}$.

   (a) (20) Let consumption be $c_i = w - R_i - i\tau$. Set up the household's problem.

   (b) (20) Using the big theorem from the lecture, solve for the share of household s_i in each location.

   (c) (20) Let the share of households in each location $s_1 = s_2 = s_3 = \frac{1}{3}$, wage $w = 5$ and the price of agricultural land $\bar{R} = 1$. Assume that the land rent at $x = 3$ is equal to $\bar{R}$. Solve for R_1, R_2 and R_3 in terms of τ.

   (d) (20) Solve for consumption in terms of τ.

   (e) (10) Plot land rent and commuting costs as a function of i. How does this compare to the monocentric city model with a continuum of locations?

2. Consider a city consisting of three locations i, j = 1,2,3. Households choose one location for work and one (maybe the same one) for their residence. Each location pays wage w and has residential rent R_j. Commute costs are iceberg, with $\tau_{ij} = 1$ if $i = j$ and $\tau_{ij} > 1$ otherwise. Each workplace-residence choice has an amenity value A_{ij}, with A_{11} normalized to 1.

   Each household type ν gets a Frechet taste shock $z_{ij}(\nu)$ for each workplace-residence pair. Shocks are i.i.d. with level parameter T and dispersion θ. Household payoffs from location choice (i,j) are $V_{ij}(\nu) = z_{ij}(\nu) A_{ij} w / \tau_{ij} R_j^\beta$, where $\beta \in$ (0,1) is the housing share of consumption.

   Suppose w = 10, (R_1,R_2,R_3) = (1,1,1), $\beta = 1/3$, $\theta = 8$, observed shares are $s_{ij} = \frac{1}{9}$ all i,j, and transportation costs are

$$\begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{bmatrix} = \begin{bmatrix} 1.00 & 1.10 & 1.21 \\ 1.10 & 1.00 & 1.10 \\ 1.21 & 1.10 & 1.00 \end{bmatrix}.$$

Note that this describes three equally spaced locations on a line with iceberg factor 1.1 to travel one position along the line.

(a) Use the available data to solve for the A_{ij}. You would usually need to solve this numerically, but this one you can solve by hand.

(b) Consider a counterfactual transportation cost matrix

$$\begin{bmatrix} \tau'_{11} & \tau'_{12} & \tau'_{13} \\ \tau'_{21} & \tau'_{22} & \tau'_{23} \\ \tau'_{31} & \tau'_{32} & \tau'_{33} \end{bmatrix} = \begin{bmatrix} 1.00 & 1.20 & 1.44 \\ 1.20 & 1.00 & 1.20 \\ 1.44 & 1.20 & 1.00 \end{bmatrix}.$$

(c) Using the values of A_{ij} that you found above, evaluate counterfactual values of s_{ij} under the new transportation cost matrix.