

①

CONSUMER SAVES

$$(a) \quad \text{MAX} \left[\int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} T^{1-\mu}$$

$$\text{s.t.} \quad \int_0^M p(i) f(i) di + p^T T = w \quad (1)$$

FIRST ORDER CONDITION IS $\forall i = \lambda p \quad j \in [0, M], T, \lambda \geq 0$ SET $p^T = 1$, W.O.L.G.

$$\left[\int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} T^{-\mu} (1-\mu) = \lambda \quad (2)$$

$$\mu \frac{\sigma}{\sigma-1} \left[\int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\mu\sigma}{\sigma-1} - 1} \frac{\sigma-1}{\sigma} f(j)^{-\frac{1}{\sigma}} T^{1-\mu} = \lambda p(j), \quad j \in [0, M] \quad (3)$$

② \rightarrow ③

$$\Rightarrow \mu \frac{\sigma}{\sigma-1} \left[\int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{-1} \frac{\sigma-1}{\sigma} f(j)^{-\frac{1}{\sigma}} T = (1-\mu) p(j), \quad j \in [0, M]$$

$$\Rightarrow \mu \left[\int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{-1} f(j)^{-\frac{1}{\sigma}} T = (1-\mu) p(j), \quad j \in [0, M] \quad (4)$$

$$(4) \Rightarrow \frac{p(k)}{p(j)} = \left[\frac{f(k)}{f(j)} \right]^{-\frac{1}{\sigma}} \quad ; \quad j, k \in [0, M]$$

$$\Rightarrow \left(\frac{p(j)}{p(k)} \right)^{\sigma} = \frac{f(k)}{f(j)} \Rightarrow f(k) = f(j) \left(\frac{p(j)}{p(k)} \right)^{\sigma} \quad (5)$$

$j, k \in [0, M]$

(2)

using (5) in (4)

$$\mu \left[\int_0^M \left[f(j) \left(\frac{p(j)}{p(i)} \right)^{\sigma} \right]^{\frac{\sigma-1}{\sigma}} di \right]^{-1} f(j)^{-\frac{1}{\sigma}} = (1-\mu) p(j)$$

$$\Rightarrow f(j)^{\frac{1}{\sigma}} f(j)^{-\frac{(\sigma-1)}{\sigma}} \left[\int_0^M \left[\frac{p(j)}{p(i)} \right]^{\sigma-1} di \right]^{-1} = \frac{1-\mu}{\mu T} p(j)$$

$$\begin{aligned} f(j)^{-1} &= \left[\int_0^M \left[\frac{p(j)}{p(i)} \right]^{\sigma-1} di \right] \frac{1-\mu}{\mu T} p(j) \\ &= p(j)^{\sigma-1} p(j) \left[\int_0^M p(i)^{1-\sigma} di \right] \frac{1-\mu}{\mu T} \\ &= p(j)^{\sigma} \int_0^M p(i)^{1-\sigma} di \frac{1-\mu}{\mu T} \end{aligned}$$

$$\Rightarrow f(j) = \frac{p(j)^{-\sigma}}{\int_0^M p(i)^{1-\sigma} di} \frac{\mu T}{1-\mu} \quad (6)$$

(6) \rightarrow (2)

$$\int_0^M \frac{p(j)^{1-\sigma}}{\left[\int_0^M p(i)^{1-\sigma} di \right]} \frac{\mu T}{1-\mu} dj + T = w$$

$$\Rightarrow \frac{\mu T}{1-\mu} + T = w$$

$$\Rightarrow \boxed{T^* = (1-\mu)w} \quad (7)$$

(3)

(7) \rightarrow (6) \Rightarrow
$$f^*(y) = \frac{P(y)^{-\sigma}}{\int_0^M P(i)^{-\sigma} di} \text{ MW}$$

$$y \in [0, M]$$

DEFINE $P = \left[\int_0^M P(i)^{-\sigma} di \right]^{-\frac{1}{\sigma-1}}$ "C.F.S. PRICE INDEX"

$\Rightarrow P^{-(\sigma-1)} = \left[\int_0^M P(i)^{1-\sigma} di \right]$ (9)

(9) \rightarrow (8) $\Rightarrow f^*(y) = \frac{P(y)^{-\sigma}}{P^{-(\sigma-1)}} \text{ MW}$

\Rightarrow
$$f^*(y) = P^{\sigma-1} P(y)^{-\sigma} \text{ MW}$$
 (10) (c.f. FUJITA + THISSER eqn 9.4)

STRICTLY, IF $K \subset [0, M]$ MEASURE ZERO AND $f: K \rightarrow \mathbb{R}$ THEN $f^* + f$ IS ALSO SOLVES THE CONSUMERS PROBLEM.



(4)

PROFITS FOR FIRM j ARE

$$\begin{aligned}\pi &= P(j) q(j) - A - B q(j) \\ &= (P(j) - B) q(j) - A\end{aligned}$$

From (10)

$$\begin{aligned}\Rightarrow \pi &= (P(j) - B) \frac{M W}{P^{1-\sigma}} P(j)^{\sigma} - A && \text{(RECALL, FIRMS ARE MONOPOLISTS)} \\ &= \left[P(j)^{1-\sigma} - B P(j)^{-\sigma} \right] \frac{M W}{P^{1-\sigma}} - A\end{aligned}$$

$$\frac{d\pi}{dP(j)} = \frac{M W}{P^{1-\sigma}} \left[(1-\sigma) P(j)^{-\sigma} + B \sigma P(j)^{-\sigma-1} \right] = 0$$

$$\Rightarrow (1-\sigma) + B \sigma P(j)^{-1} = 0$$

$$(1-\sigma) = -B \sigma P(j)^{-1}$$

$$\Rightarrow P^*(j) = \frac{-B \sigma}{1-\sigma}$$

(NEED $\sigma > 1$) (11)

USING (11) IN (8)

$$q^*(j) = \frac{\left[\frac{-B \sigma}{1-\sigma} \right]^{\sigma}}{M \left[\frac{-B \sigma}{1-\sigma} \right]^{1-\sigma}} = \frac{1-\sigma}{-M B \sigma} = \frac{\sigma-1}{M B \sigma}, \quad q \in [0, \infty]$$

(12)

(5)

WITH FREE ENTRY OF FIRMS $\pi = 0$

$$\Rightarrow Pq - A - Bq = 0$$

USING (11) + (12)

$$\Rightarrow \left(\frac{-Bq}{1-\sigma} \right) \left(\frac{\sigma-1}{MBR} \right) - A - B \left(\frac{\sigma-1}{MBR} \right) = 0$$

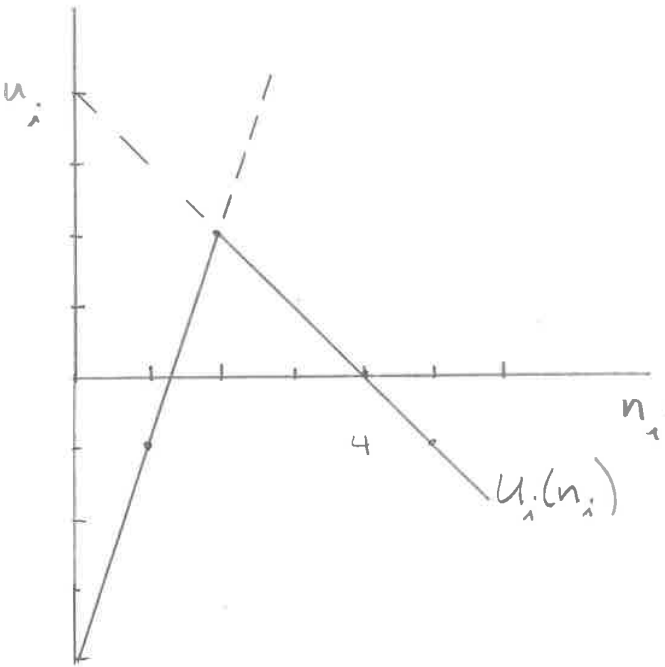
$$\Rightarrow \frac{\sigma}{M\sigma} - A - \frac{\sigma-1}{M\sigma} = 0$$

$$\Rightarrow \frac{1}{M\sigma} - A = 0, \quad (13)$$

HERE, UNLIKE F.T ON KRUGMANI, WAGE IS EXOGENOUS, SO WE ONLY GET ZERO ENTRY CONDITION TO HOLD IF PARAMETERS HAPPEN TO SATISFY (13).

(b) SEE EQ. (11) ABOVE - THIS IS CONSTANT MARK-UP BEYOND MARGINAL COST (= B)

(c) YOU CANNOT VERIFY THIS STATEMENT WITHOUT ENDOGENIZING THE WAGE.



① Suppose $N=5$. There are six candidate equilibria,
 (a) 0, 1, 2, 3, 4, 5 people choose urban.

If 0 people choose urban, then everyone gets $\bar{u}=0$ and unilateral deviations allow $u(1)=-1$, so $n_i=0$ is Nash.

If $n=4$, then $u_i(4)=\bar{u}=0$. An urban dweller who deviates to non-urban gets 0. This is not a beneficial deviation. A rural dweller who deviates to urban gets $u_i(5)=-1$, also not a rational deviation.

\therefore Any strategies s.t. $n_i \in \{0, 4\}$ is a Nash Equil.

If $n_i \in \{1, 5\}$ then $u = -1 < \bar{u}$ so deviations to non-urban are rational.

If $n = 2$ then $u_i = 2$ and a deviation to urban

(2)

GIVES PA-OFF $U(3) = 1 > \bar{U}$, SO $n_i = 2$ IS NOT NASH.

IF $n_i = 3$, THEN DEVIATIONS TO URBAN GIVE $U(4) = 0 = \bar{U}$ AND DEVIATIONS FROM URBAN GIVE $\bar{U} = 0 < 1 = U(3)$.
 \Rightarrow STRATEGIES S.T. $n = 3$ ARE NASH.

THIS, STRATEGIES, ARE NASH $\Leftrightarrow n \in \{0, 3, 4\}$.

(b) FOR MIXED STRATEGIES, EACH AGENT CHOOSES $p_i \in [0, 1]$ WE CAN USE THIS TO WRITE DOWN THE PROBABILITIES OF EACH CONFIGURATION.

n_i	PROB	$U(n_i)$
0	$(1-p)^5$	-4
1	$p(1-p)^4$	-1
2	$p^2(1-p)^3$	2
3	$p^3(1-p)^2$	1
4	$p^4(1-p)$	0
5	p^5	-1

FOR THE PURPOSE OF CALCULATING MIXED EQUIL, WE NEED TO KNOW THE EFFECT ON PA-OFFS OF CHANGING p FOR ONE AGENT, HOLDING OTHERS FIXED AT p' .

(3)

IF ALL AGENTS BUT 1 CHOOSE P , AND 1 CHOOSES P' , THEN WHAT WE CARE ABOUT IS THE DISTRIBUTION OF EVERYONE ELSE'S MOVES. LET n' DENOTE # OF PLAYERS, $\neq 1$, CHOOSING UNBANK THEN

n'	$PR(n')$
0	$(1-p)^4$
1	$p(1-p)^3$
2	$p^2(1-p)^2$
3	$p^3(1-p)$
4	p^4

THEN, IF 1 CHOOSES P' ,

$$E(\pi_i) = (1-p') \cdot 0 + \quad (1 \text{ is, BANK})$$

$$p' \left[-1(1-p)^4 + 2(1-p)^3 p + 1(1-p)^2 p^2 \right. \\ \left. + 0 \cdot (1-p)p^3 - 1p^4 \right] \quad (*)$$

WE NEED TO FIND p' SUCH THAT

$$\frac{d(E\pi_i)}{dp'} = 0 \quad \text{AND} \quad p' = p$$

WHERE THE SECOND CONDITION IS BIC WE'VE RESTRICTED ATTENTION TO SYMMETRIC EQUIL.

BY INSPECTION OF (*) WE NEED THE EXPRESSION IN BRACKETS TO BE ZERO, SO WE WILL NEED TO CHOOSE p SO THAT IT IS A ZERO OF THIS 4TH ORDER POLYNOMIAL. \Rightarrow EXPECT AS MANY AS 4 ROOTS.

(4)

(c) THE PARETO OPTIMAL OUTCOMES ARE $n \in \{2, 3\}$
 FOR EITHER URBAN POPULATION, THERE IS NO WAY
 TO MAKE ONE PERSON BETTER W/O MAKING
 ANOTHER WORSE.

(d) WE NEED $n_1 + n_2 \leq 5$.

(1) $(n_1, n_2) \in \{(0, 0), (0, 4), (4, 0)\}$ ARE
 ALL NASH. IN THESE EQUIL, EVERYONE GETS \bar{u} .

(2) $(n_1, n_2) \in \{(2, 3), (3, 2)\}$

HERE THE URBAN PA-OFF IS POSITIVE, SO
 NO ONE MOVES TO RURAL. NO ONE WANTS
 TO MOVE FROM THE SMALL TO THE BIG CITY,
 THIS IS STRICTLY WORSE. MOVING FROM SMALL CITY
 TO BIG DOES NOT CHANGE PA-OFF FOR MARGINAL
 AGENT.

(3) NO OUTCOME WITH $n_i \in \{1, 5\}$ IS NASH
 B/C ONE AGENT CAN DEVIATE TO RURAL FOR
 $0 > -1$.

(4) NO OUTCOME $n_i = 2$ IS AN EQUILIBRIUM
 UNLESS $n_1 + n_2 = 5$. OTHERWISE RURAL
 AGENTS WILL DEVIATE

(5) $(n_1, n_2) \in \{(0, 3), (3, 0)\}$ ARE ALSO EQUIL.
 URBAN RESIDENTS DON'T WANT TO MOVE, AND
 RURAL AGENTS WHO MOVE TO THE CITY ARE
 INDIFFERENT.

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Summing up, we have

		n_1					
		0	1	2	3	4	5
n_2	0	*	X	X	*	*	X
	1	X	X	X	X	X	X
	2	X	X	X	*	X	X
	3	*	X	*	X	X	X
	4	*	X	X	X	X	X
	5	X	X	X	X	X	X

 $\Rightarrow n_1 + n_2 > 5$

* \Rightarrow NASH

X \Rightarrow NOT NASH

 \Rightarrow T.O.

Therefore, NASH EQUIL IS CONSISTENT WITH TWO CITIES OF ABOUT OPTIMAL SIZE AND AN EMPTY COUNTRYSIDE, CAL WITH ONE CITY LARGER THAN OPTIMAL WITH 1 OR 2 STILL RURAL.

(e) ADDING ANOTHER CITY WOULD NOT QUALITATIVELY CHANGE THE EQUILIBRIUM OUTCOMES, IT WOULD JUST CREATE INDETERMINACY ABOUT WHICH (0) CITIES ARE OCCUPIED. THIS SEEMS SPECIAL TO THE CASE $N=5$.