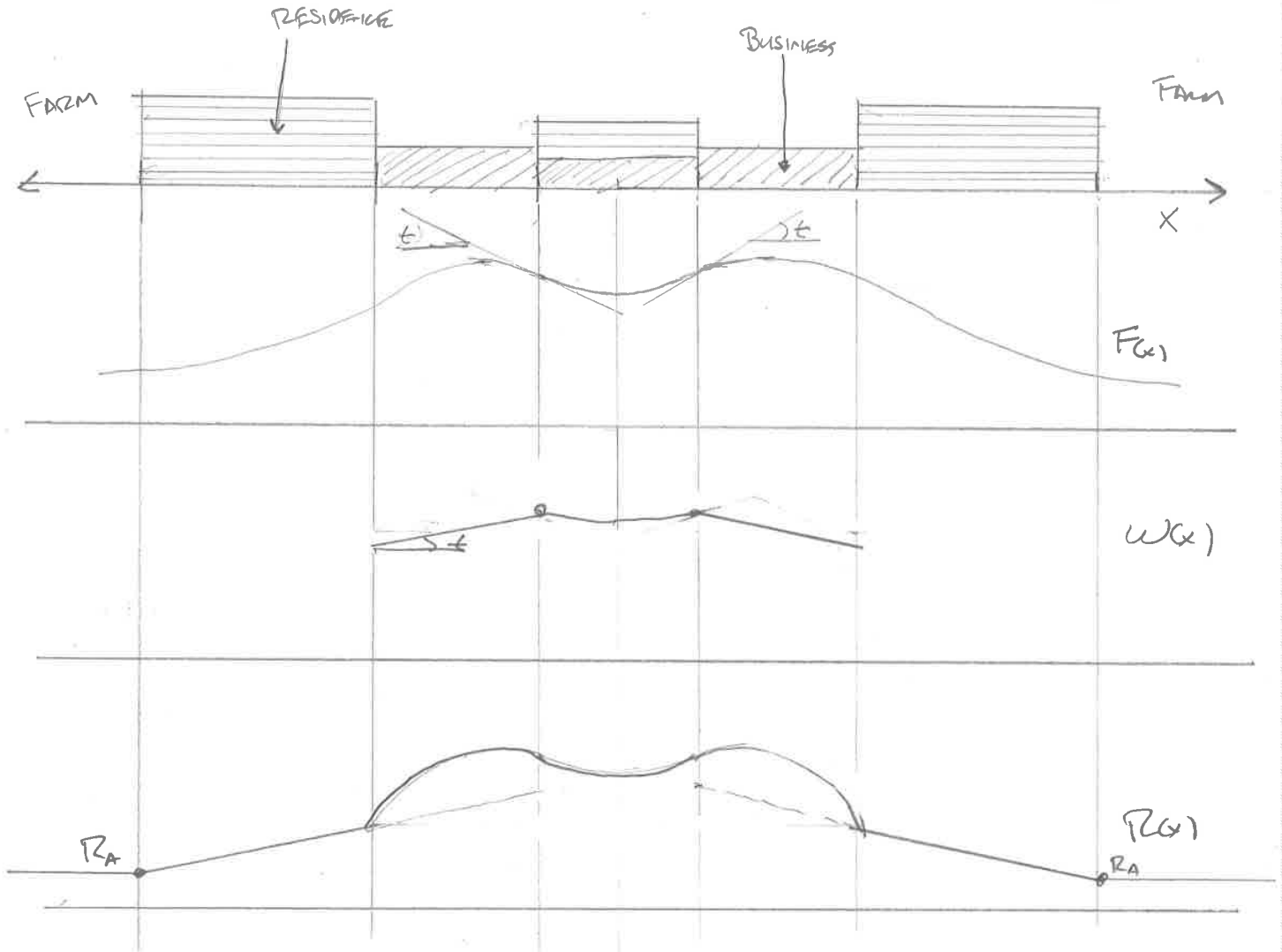


FUSITA + OGAWA PARTIALLY MIXED EQUIL



1. CENTER IS MIXED \Rightarrow WORK WHERE YOU LIVE
 \Rightarrow SLOPE OR WAGE GRADIENT LESS THAN t .
2. PEOPLE IN ADJACENT BUSINESS DISTRICTS COMMUTE FROM OUTSIDE \Rightarrow WAGE GRADIENT HAS SLOPE t
 AND RENT GRADIENT ALWAYS ABOVE t_x .

①

- $S \in [S_1, S_2]$ ~ AMENITY
- X ~ NUMERAIRES CONSUMPTION GOOD
- l^c ~ RESIDENTIAL LAND
- K ~ UTILITY LEVEL
- W, I ~ WAGE, NON-WAGE INCOME
- r ~ LAND RENT

N ~ LABOR = PEOPLE

l^p ~ PRODUCTIVE LAND

$$X = f(N, l^p; s)$$

~ CRS.

$$\textcircled{1} L = l^c + l^p$$

FREE MOBILITY

- CONSUMERS SOLVE: $\text{MAX } U(X, l^c; s)$
 S.T. $X + r l^c = W + I$ \Rightarrow $\textcircled{2} V(W, r; s) = K$

- FIRMS HAVE UNIT COST FUNCTION

$$\textcircled{3} C(W, r; s) = 1$$

↑ FREE ENTRY + $\pi = 0 + P_X = 1$.

WITH CRS \Rightarrow $\textcircled{4} C_r = \frac{l^p}{X}$, $\textcircled{5} C_w = \frac{N}{X}$

AN EQUILIBRIUM MUST SATISFY $\textcircled{1}$ - $\textcircled{5}$.

TOTAL DIF $\textcircled{2} + \textcircled{3} \Rightarrow V_w \frac{dw}{ds} + V_r \frac{dr}{ds} = +V_s \textcircled{6}$

$$C_w \frac{dw}{ds} + C_r \frac{dr}{ds} = -C_s \textcircled{7}$$

SOLVE $\textcircled{6} + \textcircled{7}$ FOR $\frac{dr}{ds} = \frac{-V_w C_s + V_s C_w}{V_w C_r - V_r C_w} \equiv \frac{V_w C_s - V_s C_w}{\Delta}$

SOLVE $\textcircled{6} + \textcircled{7}$ FOR $\frac{dw}{ds} = \frac{-V_s C_r + V_r C_s}{V_w C_r - V_r C_w} \equiv \frac{V_s C_r - V_r C_s}{\Delta}$

(2)

USING (4) + (5) WE HAVE

$$\Delta = -V_r C_w + V_w C_r = -V_r \frac{N}{X} + V_w \frac{l^P}{X}$$

$$= V_w \left[-\frac{V_r}{V_w} \frac{N}{X} + \frac{l^P}{X} \right]$$

USING RYD'S IDENTITY,

$$= V_w \left[+l^c \frac{N}{X} + \frac{l^P}{X} \right]$$

$$= +V_w \left[\frac{l^c N + l^P}{X} \right]$$

$$= \frac{+V_w L}{X}$$

FROM USING CRS OF C(.) IN (7)

$$\Rightarrow -C_s = \frac{N}{X} \frac{dw}{ds} + \frac{l^P}{X} \frac{dr}{ds}$$

$$\Rightarrow C_s = - \left[\frac{N}{X} \frac{dw}{ds} + \frac{l^P}{X} \frac{dr}{ds} \right] \quad (8)$$

USING CRS OF C(.) IN (6)

$$\Rightarrow \frac{dw}{ds} + \frac{V_r}{V_w} \frac{dr}{ds} = \frac{V_s}{V_w} \quad (\text{USE RYD'S IDENTITY})$$

$$\Rightarrow \frac{dw}{ds} = l^c \frac{dr}{ds} + \frac{V_s}{V_w} \quad \left[\frac{V_r}{V_w} = -l^c \right]$$

$$\Rightarrow \frac{dw}{ds} = l^c \left[\frac{-V_w C_s + V_s C_w}{\Delta} \right] + \frac{V_s}{V_w}$$

$$\Rightarrow \frac{dw}{ds} = \frac{+X l^c}{V_w L} \left[-V_w C_s + V_s \frac{N}{X} \right] + \frac{V_s}{V_w}$$

→

(8)

$$\Rightarrow \frac{dw}{ds} = -\frac{x l^c}{L} c_s - \frac{N l^c}{L} \frac{v_s}{v_w} + \frac{v_s}{v_w}$$

$$P_s^* \equiv \frac{v_s}{v_w} \Rightarrow \frac{dw}{ds} = -\frac{x l^c}{L} c_s - \frac{N l^c}{L} P_s^* + P_s^*$$

$$\frac{dw}{ds} = -\frac{x l^c}{L} c_s + \frac{L - N l^c}{L} P_s^*$$

$$\frac{dw}{ds} = -\frac{x l^c}{L} c_s + \frac{l^p}{L} P_s^* \quad (9)$$

USING (8) TO SUBSTITUTE FOR c_s IN (9)

$$\frac{dw}{ds} = +\frac{x l^c}{L} \left[c_w \frac{dw}{ds} + c_r \frac{dr}{ds} \right] + \frac{l^p}{L} P_s^*$$

USING CRS

$$= +\frac{x l^c}{L} \left[\frac{N}{x} \frac{dw}{ds} + \frac{l^p}{x} \frac{dr}{ds} \right] + \frac{l^p}{L} P_s^*$$

$$\frac{l^p}{L} P_s^* = \frac{dw}{ds} - \frac{x l^c}{L} \left[\frac{N}{x} \frac{dw}{ds} + \frac{l^p}{x} \frac{dr}{ds} \right]$$

$$\frac{l^p}{L} P_s^* = \frac{dw}{ds} - \frac{l^c N}{L} \frac{dw}{ds} - \frac{l^c l^p}{L} \frac{dr}{ds}$$

$$P_s^* = \left(\frac{L}{l^p} + \frac{l^c N}{l^p} \right) \frac{dw}{ds} - l^c \frac{dr}{ds}$$

$$\Rightarrow \boxed{P_s^* = \frac{dw}{ds} - l^c \frac{dr}{ds}} \quad \square$$