$$U(\theta) = \begin{cases} W - R & \text{IF } \theta \text{ IN CITY} \\ \theta & \text{ELSE} \end{cases}$$

$\theta \in [0, \Theta]$.

(a) IN EQUILIBRIUM, H.H. CHOOSE CITY IFF $W - R \geq \theta$.
IF $\Theta > 1 > W$, THEN EVEN IF $R = 0$ WE HAVE H.H. WITH $\theta > W$ OUTSIDE THE CITY $\Rightarrow$ LESS THAN MEASURE 1 OF LAND OCCUPIED IN CITY FOR ANY $R \geq 0 \Rightarrow$ MARGINAL CITY LAND IS VACANT $\Rightarrow R = 0$.

IF $\Theta > W > 1$ THEN $R^* = W - 1$ IS EQUILIBRIUM RENT. IN THIS CASE $\theta = W - R^* = 1$
SO THE CITY IS FULLY OCCUPIED AND NO ONE WANTS TO MOVE OR CHANGE R.

(b) IF $\Theta > 1 > W$, THEN $R = 0$ AND

$\theta \in [0, W]$ IN CITY, $\theta \in (W, \Theta)$ OUT.

SINCE RENT IS ZERO, LAND RENT IS ZERO. BUT

$$CS = \int_0^W (W - \theta)\, d\theta = \frac{1}{2} W^2$$

IF $\Theta > W > 1$ THEN $R = W - 1$ AND 1 UNIT OF HH OCCUPY CITY

$$CS = \int_0^1 W - R - \theta \, d\theta = \int_0^1 1 - \theta \, d\theta = \frac{1}{2}$$

$$\text{RENT} = (W - 1) \cdot 1 = W - 1$$

(c) WITH HETEROGENEOUS OUTSIDE OPTIONS, WE NEED TO WORRY ABOUT CS AS WELL AS RENT.

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(2) MAX $u(c)$

$w = c + R(x) + 2tx$

(a) LET $c^* = u^{-1}(\bar{u})$.

THEN WE HAVE $R(x) + 2tx = w - c^*$ ① $\forall x \in [-\bar{x}, \bar{x}]$

IN PARTICULAR, $\bar{R} + 2t\bar{x} = w - c^*$

$$\Rightarrow \quad \bar{x} = \frac{w - c^* - \bar{R}}{2t}$$

(b)(i) FROM ABOVE, WHEN w INCREASES TO w', WE HAVE

$$\bar{x} = \frac{w - c^* - \bar{R}}{2t}$$

AND $$\bar{x}' = \bar{x} + \frac{w' - w}{2t}$$

From ①

$$R(x) = \begin{cases} w - c^* - 2tx & x \in [-\bar{x}, \bar{x}] \\ \bar{R} & \text{ELSE} \end{cases}$$

$$R'(x) \begin{cases} w' - c^* - 2tx & x \in [-\bar{x}', \bar{x}'] \\ \bar{R} & \text{ELSE.} \end{cases}$$

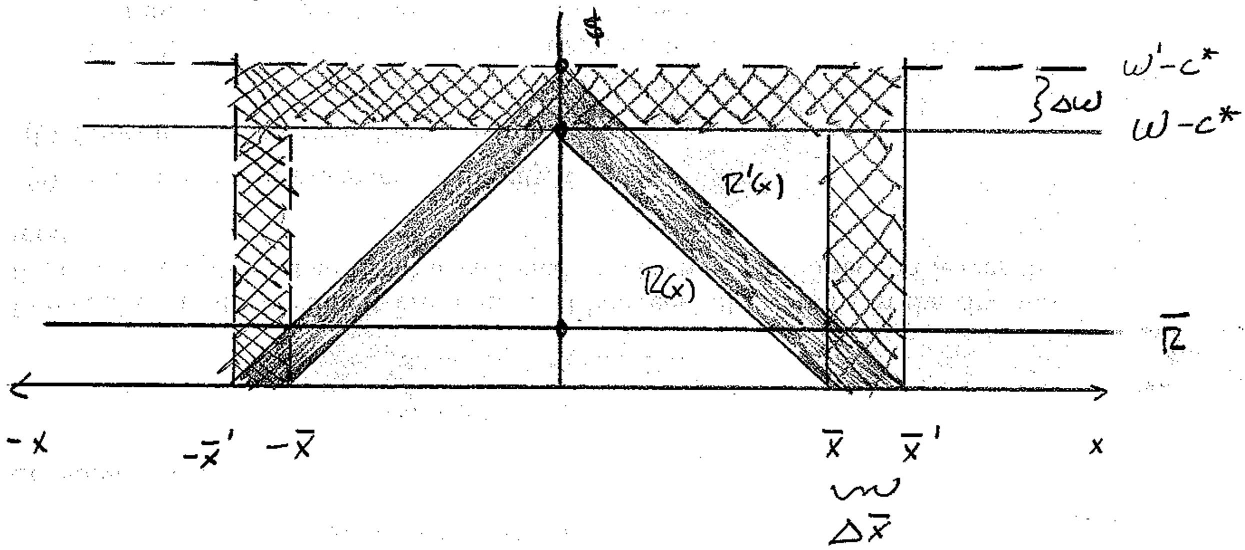
Thus

$$R'(x) - R(x) = \begin{cases} w' - w & x \in [-\bar{x}, \bar{x}] \\ w' - c^* - 2tx - \bar{R} & x \in \left\{[-\bar{x}', -\bar{x}] \cup [\bar{x}, \bar{x}']\right\} \\ 0 & \text{ELSE} \end{cases}$$

$\longrightarrow$

so $$\int_{-\infty}^{\infty} R' - R\,dx = 2\left[(w'-w)\,\bar{x} - \frac{1}{2}\left(w - c^* - 2t\bar{x}' - \bar{R}\right)\left(\bar{x}' - \bar{x}\right)\right]$$

(ii) IF NEW MIGRANTS WERE ALSO PAID w, THEN AGG. WAGE INCREASE IS $2\bar{x}(w'-w)$.



THE HATCHED AREA GIVES CHANGE IN AGGREGATE WAGE INCOME RESULTING FROM Δw.

THE SHADED AREA GIVES INCREASE IN AGGREGATE LAND RENT FROM Δw

THE INCREASE IN WAGES IS LARGER THAN THE INCREASE IN RENT BY ABOUT $2\Delta\bar{x}\, w'$.

HOWEVER, THE INCREASE IN WAGES NET OF COMMUTING IS COMPLETELY CAPITALIZED INTO RENT.