

EC2410-Spring 2019 Problem Set 1

(Updated 8 January 2020)

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1. Consider a city with measure one of land. All city residents receive a wage w and consume land inelastically so that measure 1 of residents occupies measure 1 of land, i.e., fills the city up. Residents pay land rent $R \geq 0$ and derive utility from consumption. The set of potential city residents is the set $[0, \Theta]$, with measure Θ and is indexed by θ . Agent θ 's utility is,

$$u(\theta) = \begin{cases} w - R & \text{if } \theta \text{ in city} \\ \theta & \text{else} \end{cases}$$

That is, agents get utility from consuming $w - R$ in the city, and an idiosyncratic reservation value outside the city. Consider two cases, $\Theta \geq 1 > w$ and $\Theta \geq w > 1$. Land markets are perfectly competitive.

- (a) Characterize a free mobility equilibrium for this economy, and in particular, find land rent for all locations in the city.
 - (b) Calculate aggregate land rent and consumers' surplus in equilibrium.
 - (c) Is land rent as interesting a measure of welfare in this model as in the linear city model? Explain briefly.
2. Consider the linear city model developed in class. Recall the following notation:

$l = 1 \sim$ unit land consumption for all residents

$c \sim$ composite consumption

$x \sim$ distance to center

$R(x) \sim$ unit land rent

$t \sim$ unit cost of transportation

$w \sim$ fixed wage paid to all workers at city center

$\bar{u} \sim$ reservation utility level

$\bar{R} \sim$ agricultural land rent

Consumers solve

$$\begin{aligned} \max_x & u(c) \\ \text{s.t. } & w = c + R(x) + 2tx. \end{aligned}$$

In equilibrium all consumers are indifferent between all locations in the city and their outside option. Land rent is collected by absentee landlords and leaves the model.

- (a) Find the equilibrium extent of the city, \bar{x} .
- (b) Consider an increase in the wage from w to w' .
 - i. Calculate the resulting change in aggregate land rent.
 - ii. Calculate the resulting change in aggregate wage income. Assume that migrants' wages are also w before they move.
 - iii. Draw a graph to illustrate both quantities.
 - iv. Is an infinitesimal wage increase 'completely capitalized' into land rent? Explain briefly.