Foundations of Cities

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Introduction I

How do people arrange themselves when they are free to choose work and residence locations, when commuting is costly, and when some economic mechanism rewards the concentration of employment?

Equilibrium must satisfy:

- Bookkeeping: Everyone lives and works somewhere, land markets clear.
- Households optimize and choose their favorite locations.
- Production is competitive and profits are zero everywhere.

We are used to thinking about the first two. The third is less well understood.

Really high level conclusion: When the location of production is endogenous, the details of the production process have qualitative implications for equilibrium.

Introduction II

We consider and economy where

- Spillovers, returns to scale and first nature all operate.
- Geography is simple and discrete (three locations on a line).
- Households have heterogenous preferences over workplace-residence pairs.

Equilibria contradict many of my priors,

- Corner equilibria are pervasive.
- First nature is boring.
- Returns to scale is a dispersion force when it is strong enough.
- Spillovers can act as a dispersion force when spillovers and returns to scale are small.
- Preference heterogeneity is an agglomeration force.
- Multiple equilibria are pervasive and (can be) invisible to numerical methods.
- Stability rules out corner equilibria.

Introduction III

This is important because

- Having a theory of cities is important, a lot of people live in them. The 'Mills Agenda' is substantially incomplete.
- The relationship between density and returns to scale and spillovers is more subtle than we understood. This has implications for empirical methods.
- Our framework looks a lot like QSM, and so we are developing intuition about how these models work. Do we believe/can we test the mechanisms behind counterfactuals? It's hard to know, unless we understand what mechanisms are at work.

Literature I

- *Urban Economics*, continuous space, homogenous agents, simple geographies;
  - Ogawa and Fujita (1980), additive spillovers, heroic simplifying assumption, analytic solutions.
  - Fujita and Ogawa (1982), exponential decay of spillovers, heroic simplifying assumptions, limited numerical results.
  - Lucas and Rossi-Hansberg (2002), exponential decay of spillovers and global IRS, existence and uniqueness if global IRS is weak, numerical solutions.
- *QSM*, e.g. Ahlfeldt et al. (2015)
  - Heterogenous agents
  - Discrete, empirically founded geographies.
  - Returns to scale and/or spillovers, and first nature.
  - Existence/uniqueness for weak returns to scale, numerical solutions.

We have little understanding about qualitative implications of first nature, returns to scale or spillovers, especially when they are strong enough to allow multiple equilibria.

Model I

Preferences/Indirect utility function:

$$V_{ij}(\nu) = z_{ij}(\nu) B_i D_j \frac{W_j}{\tau_{ij} R_i^{\beta}}$$

$$\begin{aligned} i, j &\sim \text{workplace, residence} \\ z(\nu) &\sim \text{ Frechet taste parameter for type } \nu \text{, dispersion } \varepsilon \\ W &\sim \text{ Wage} \\ \tau &\sim \text{ icerberg commuting} \\ R &\sim \text{Land rent (=housing)} \\ \beta &\sim \text{housing share in Cobb-Douglas utility} \\ B,\ D &\sim \text{Residential, Workplace amenities} \end{aligned}$$

This is conventional.

Model II

Utility maximization plus z Frechet implies that

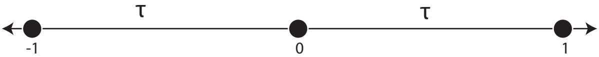
$$s_{ij} = \frac{\left[B_i D_j W_j / \left(\tau_{ij} R_i^{\beta}\right)\right]^{\varepsilon}}{\sum_r \sum_s \left[B_r D_s W_s / \left(\tau_{rs} R_r^{\beta}\right)\right]^{\varepsilon}}$$

for s_{ij} share of population with ij as their favorite outcome.

Model III

Geography and commuting:

- Three locations, $i, j \in \{-1, 0, 1\}$.



- $\tau \geq 1$ is unit (iceberg) commute cost, so

$$\begin{pmatrix} \tau_{-1,-1} & \tau_{-1,0} & \tau_{-1,1} \\ \tau_{0,-1} & \tau_{0,0} & \tau_{0,1} \\ \tau_{1,-1} & \tau_{1,0} & \tau_{1,1} \end{pmatrix} = \begin{pmatrix} 1 & \tau & \tau^2 \\ \tau & 1 & \tau \\ \tau^2 & \tau & 1 \end{pmatrix}$$

- This is (1) the simplest geography where we can talk about ‘central’ and ‘peripheral’ (2) the discrete analog of the canonical monocentric city (3) tractable and transparent.

Model IV

Notation:

- Restrict attention to symmetric cities, i.e., $W_1 = W_{-1}$ etc.
- $\phi \equiv \tau^{-\varepsilon}$ is *spatial discount factor*.
  NB: $\phi \to 0$ when $\tau \to \infty$ or $\varepsilon \to \infty$
  ($\varepsilon \to \infty \implies$ no heterogeneity).
- M_i is residence at i. L_j is employment at j.
- H_i is housing at i. N_j is commercial land at j.
- $w \equiv W_0/W_1$, $r = R_0/R_1$, and so on.
  We solve the model in ratios.
- $\rho \equiv \left(br^{-\beta}\right)^{\varepsilon}$ *inverse amenity-adjusted relative rent*.
- $\omega \equiv (dw)^{\epsilon}$ *amenity-adjusted relative wage*.

Model V

Utility maximization (alone) implies

$$\begin{pmatrix} s_{11} & s_{10} & s_{1-1} \\ s_{01} & s_{00} & s_{0-1} \\ s_{-11} & s_{-10} & s_{-1-1} \end{pmatrix} = \frac{1}{\rho\omega + 2\phi(\rho+\omega) + 2(1+\phi^2)} \begin{pmatrix} 1 & \phi\omega & \phi^2 \\ \phi\rho & \rho\omega & \phi\rho \\ \phi^2 & \phi\omega & 1 \end{pmatrix}.$$

Utility maximization lets us write any model quantity in terms of ρ and ω, e.g.,

$$M_0 = s_{00} + 2s_{01} \qquad \text{(residence at zero)}$$

$$L_1 = s_{01} + (1+\phi^2)s_{11} \quad \text{(employment at one)}$$

Model VI

Perfect competition: Firms pay land and labor their marginal revenue product.

Production

$$Y_0 = A_0 L_0^{\alpha} N_0^{1-\alpha}, \quad Y_1 = A_1 L_1^{\alpha} N_1^{1-\alpha}$$

(α is labor share)

$$A_0 = C_0 L_0^{\gamma} + 2\delta L_1, \quad A_1 = C_1 L_1^{\gamma} + \delta L_0 + \delta^2 L_1$$

$$C_j = \text{first nature}$$

$$\gamma = \text{IRS}$$

$$\delta = \text{spillovers}$$

Model VII

- First nature, $c \neq 1, \delta = 0, \gamma = 0 \Longrightarrow$

$$A_0 = C_0, \quad A_1 = C_1$$

- Local IRS, $c = 1, \delta = 0, \gamma > 0 \Longrightarrow$

$$A_0 = L_0^\gamma, \quad A_1 = L_1^\gamma$$

- Spillovers,$c = 1, \delta > 0, \gamma = 0 \Longrightarrow$

$$A_0 = 2\delta L_1, \quad A_1 = \delta L_0 + \delta^2 L_1$$

  This is a standard (discrete) potential function with exponential decay. N.B.: Lucas and Rossi-Hansberg (2002) and Ahlfeldt et al. (2015) allow for 'global returns to scale'.

Model VIII

Cost minimization,

$$\frac{W_j}{R_j} = \frac{\alpha}{1-\alpha}\frac{N_j}{L_j}.$$

Ratio of $i = 0$ to $i = 1$,

$$\frac{r}{w} = \frac{\ell}{n}.$$

Zero profit condition (unit cost = numeraire price)

$$\frac{1}{A_i}\left(\frac{W_i}{\alpha}\right)^{\alpha}\left(\frac{R_i}{1-\alpha}\right)^{1-\alpha} = 1.$$

Ratio of $i = 0$ to $i = 1$,

$$\frac{w^{\alpha} r^{1-\alpha}}{a} = 1,$$

where $a = \frac{A_0}{A_1}$ = ratio of central to peripheral TFP.

Model IX

Bookkeeping:

- Everyone works somewhere: $L_0 + 2L_1 = 1$.
- Everyone lives somewhere: $M_0 + 2M_1 = 1$.
- Land market clears: $H_1 + N_1 = 1$, $H_0 + N_0 = 1$.

Equilibrium I

Equilibrium requires: cost min., utility max., everyone lives/works somewhere, land markets clear, zero profits, and perfect competition.

Define the 'market clearing' locus, of (ρ, ω):

$$\omega^{\frac{1+\varepsilon}{\varepsilon}} = f(\rho),$$

using cost minimization, utility maximization, everyone lives/works somewhere, **land markets clear**, ~~zero profits~~, perfect competition.

Define the 'zero profit' locus of (ρ, ω):

$$\omega^{\frac{1+\varepsilon}{\varepsilon}} = g(\rho; \gamma)$$

using cost minimization, utility maximization, everyone lives/works somewhere, ~~land markets clear~~, **zero profits**, perfect competition.

Equilibrium II

Equilibrium is relative rent ρ satisfying

$$f(\rho) = g(\rho; \gamma)$$

- Solution method; find fixed points and evaluate comparative statics.
- This only works if $\delta = 0$ (no spillovers)
- f does not involve TFP. g is where the interesting behavior arises (through the zero profit condition).

Equilibrium III

For reference,

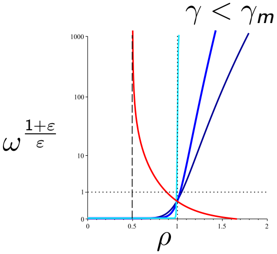
$$\omega^{\frac{1+\varepsilon}{\varepsilon}} = f(\rho) \equiv d\frac{\phi b^{\frac{1}{\beta}}\rho - 2\eta\phi\rho^{1+\frac{1}{\beta\varepsilon}} + (1+\phi^2)(1+\eta)b^{\frac{1}{\beta}}}{(1+\eta)\rho^{1+\frac{1}{\beta\varepsilon}} + 2\phi\rho^{\frac{1}{\beta\varepsilon}} - \eta\phi b^{\frac{1}{\beta}}},$$

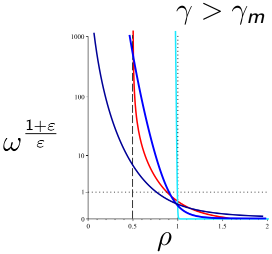
$$\omega^{\frac{1+\varepsilon}{\varepsilon}} = \begin{cases} g(\rho;\gamma) \equiv \Phi^{\frac{1}{\alpha-\gamma\varepsilon}}\rho^{\frac{\alpha\psi}{\alpha-\gamma\varepsilon}}\left(\frac{\rho+2\phi}{\phi\rho+1+\phi^2}\right)^{\frac{\gamma\varepsilon}{\alpha-\gamma\varepsilon}\frac{1+\varepsilon}{\varepsilon}} & \text{when } \delta = 0, \\ \left[\frac{1}{\Psi}a(\ell(\rho,\omega))\right]^{\frac{1+\varepsilon}{\alpha}}\rho^{\psi} & \text{when } \delta > 0, \end{cases}$$

where $\Phi \equiv c^{\varepsilon\psi\eta}d^{\alpha\varepsilon\psi\eta}b^{-\alpha\varepsilon\psi}$ and $\Psi \equiv \left(b^{\frac{1}{\eta}}d^{-1}\right)^{\alpha}$.

NB: Singularity when $\gamma = \alpha/\varepsilon$.

Equilibrium IV



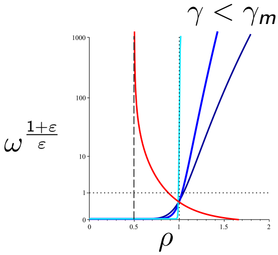


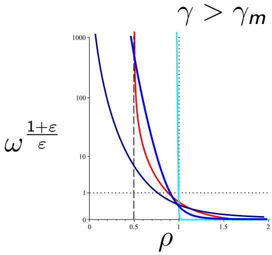
Market clearing locus, f, is red line. Wages and rents move together and f is defined only when $W_0, W_1 > 0$.

Zero profit locus, g, is blue. Step occurs when $\gamma = \gamma_m = \alpha/\varepsilon$ and all employment is in one location.

$\Longrightarrow$ an interior equilibrium exists (for $\delta = 0$).

Equilibrium V





Why does g slope up for $\gamma < \alpha/\varepsilon$, down for $\gamma > \alpha/\varepsilon$?

Consider the zero profit condition when $c = 1, \delta = 0, \gamma > 0$,

$$\frac{1}{L_i^\gamma}\left(\frac{W_i}{\alpha}\right)^\alpha \left(\frac{R_i}{1-\alpha}\right)^{1-\alpha} = 1.$$

Equilibrium with CRS

No IRS ($\gamma = 0$), no first nature ($c = d = b = 1$), no spillovers ($\delta = 0$)

Proposition:

i. A unique equilibrium exists with $W_0 < W_1$ and $R_0 > R_1$.
ii. Equilibrium becomes flat as $\varepsilon \to \infty$.
iii. $L_0 > L_1$ when land is more valuable for housing than production, production happens in land scarce center and residence in land rich periphery ($\frac{\alpha\beta}{1-\alpha} < \frac{1-\varepsilon}{\varepsilon}$).

This is surprising. *Agglomeration occurs without any conventional agglomeration force.* Why?

Let $V = W/R^\beta$ all i, j. Household's discrete choice problem is

$$\max_{ij} \left\{ \begin{array}{ccc} z_{-1,-1}V, & \frac{z_{-1,0}}{\tau}V, & \frac{z_{-1,1}}{\tau^2}V \\ \frac{z_{0,-1}}{\tau}V, & z_{0,0}V, & \frac{z_{0,1}}{\tau}V \\ \frac{z_{1,-1}}{\tau^2}V, & \frac{z_{1,0}}{\tau}V, & z_{1,1}V \end{array} \right\}.$$

Restrict households to all choose a central residence:

$$E\left(\max\left\{\frac{z_{0,-1}}{\tau}V,\ z_{0,0}V,\ \frac{z_{0,1}}{\tau}V\right\}\right) = \Gamma\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(1+\frac{2}{\tau^\varepsilon}\right)^{1/\varepsilon} V.$$

Restrict households to choose a peripheral residence:

$$E\left(\max\left\{z_{-1,-1}V,\ \frac{z_{-1,0}}{\tau}V,\ \frac{z_{-1,1}}{\tau^2}V\right\}\right) = \Gamma\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(1+\frac{1}{\tau^\varepsilon}+\frac{1}{\tau^{2\varepsilon}}\right)^{1/\varepsilon} V.$$

The first is larger than the second.

Preference heterogeneity creates an average preference for central work and residence, just like Armington/Ricardian trade advantages the center.

Equilibrium with first nature productivity

($c \neq 1$), CRS ($\gamma = 0$),no spillovers ($\delta = 0$)

Proposition:

i. As C_0/C_1 increases, so does L_0/L_1 and conversely.

ii. There exists C_0/C_1 to rationalize any L_0/L_1.

This is just what you would expect. Employment concentrates in places with first nature advantages.

Equilibrium with first nature, small spillovers, and IRS I

Proposition:

i. For $C_0/C_1 > \widehat{c}$ and $\gamma = 0$, a small *increase* in γ *increases* central employment. Conversely if $C_0/C_1 < \widetilde{c}$

ii. For $C_0/C_1 > \widetilde{c}$ and $\delta = 0$, a small *increase* in δ *decreases* central employment. Conversely if $C_0/C_1 < \widetilde{c}$

iii. $\widetilde{c} \neq \widehat{c}$.

This is surprising.

- IRS and spillovers often have opposite effects.
- Spillovers are a dispersion force in parts of the parameter space. Why? e.g., $A_1 = C_1 + \delta L_0 + \delta^2 L_1$
- Spillovers and IRS are not interchangeable ideas.
- This is a different part of the parameter space from, e.g., Fujita and Ogawa (1982) or Lucas and Rossi-Hansberg (2002).

Equilibrium with IRS, no spillovers or first nature

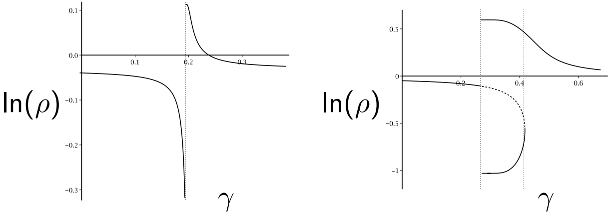
Proposition:

i. At least one interior equilibrium always exists.
ii. Corner equilibria with zero employment always exist but households always reside everywhere.

Corners are surprising. Support of workplace-residence taste shocks in unbounded. How can we have a corner solution?

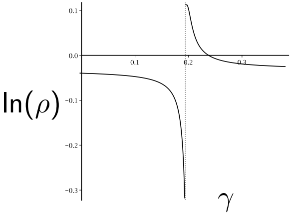
With $\gamma > 0$, $A_i = L_i^\gamma$. So TFP is zero if $L_i = 0 \Longrightarrow$ marginal revenue product of labor is identically zero, and no one wants to move.

Low commute cost and land productivity | High commute cost or land productivity

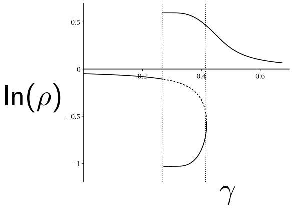


- Equilibrium is discontinuous at γ_m or γ_s.
- Equilibrium is always unique for $\gamma < \gamma_m$ and $\gamma > \gamma_s$.

Low commute cost and land productivity



High commute cost or land productivity



- $\gamma \uparrow \Longrightarrow$ employment concentrates if $\gamma < \gamma_m$.
- $\gamma \uparrow \Longrightarrow$ employment disperses if $\gamma_s < \gamma$.
- Is the threshold $\gamma_m = \alpha/\varepsilon$ empirically relevant? α is the labor share ≈ 0.5 and ε is preference dispersion $\in (3, 9)$.

This is surprising.

IRS can be a dispersion force rather than an agglomeration force. Why? Look at zero profit condition.

Stability

- We would like to use ‘stability’ as an equilibrium refinement to give us a basis for ignoring some of the multiple equilibria.
- ‘iterative stability’ is a natural criteria, i.e., ‘a fixed point algorithm will find it’.

- Given, e.g.,

$$f(\rho) = g(\rho)$$

then we want a fixed point of

$$\rho = f^{-1}(g(\rho)) = h(\rho)$$

.

- Such fixed points are 'iteratively stable' if

$$|h'| < 1$$

at the fixed point.

- Problem #1:

$$\rho = g^{-1}(h(\rho)) = \widetilde{h}(\rho)$$

has the same fixed points and opposite stability properties. Algebra matters.

- Problem #2: Solutions of

$$\widetilde{h}(\rho) = \frac{h(\rho) - (1-\theta)\rho}{\theta} = \rho$$

  are also solutions of $f(\rho) = g(\rho)$ and by choosing θ appropriately, we can change the stability properties of any fixed point. Algebra matters.

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$\Longrightarrow$ (1) iterative stability is not well defined. (2) iterative methods cannot (reliably) search for multiple equilibria. (3) 'using lots of starting values' does not respond to this problem.

We could also specify and explicit dynamic adjustment process, e.g., Krugman (1991). But this is (1) ad hoc, and (2), probably intractable.

Define *stability* 'like' trembling hand perfection: 'if a small measure of people deviate and don't want to return, equilibrium is unstable'.

This is (1) static (like our model) (2) tractable.

Corner and near-corner equilibria are always unstable.

Unstable equilibria are 'dashed lines' in figure.

Conclusion I

How do people arrange themselves when they are free to choose work and residence locations, when commuting is costly, and when some economic mechanism rewards the concentration of employment?

Some of our findings are surprising

- Corners are pervasive.
- Preference heterogeneity is an agglomeration force.
- IRS can act as a dispersion force.
- Spillovers can act as a dispersion force.

Conclusion II

Why were my priors so wrong? I had never considered the implications of the zero profit constraint.

Is this intuition general? Conjecture: much of it extends to any geography where the notion of 'center' is well defined, e.g., not a ring.

What are the implications for estimating spillovers and returns to scale? e.g., $\ln(\text{wage}) = A + B \ln(\text{density}) + C \ln(\text{Area}) + \epsilon$?

What are the implications for specifying and interpreting QSM models?

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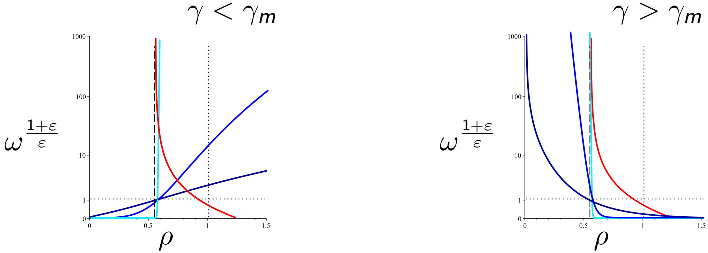

$\gamma < \gamma_m$
$\omega^{\frac{1+\varepsilon}{\varepsilon}}$
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$\gamma > \gamma_m$
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