Spatial equilibrium, capitalization and welfare: Three examples

Lecture notes #1: EC2410
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Consider two adjacent houses, 1 and 2, identical in every regard, except that in the front yard of house 1 one lives an apple tree bearing 1$/yr of apples, while the front yard of house 2 is empty. Imagine that both houses are for sale and that a number of buyers are interested in both nearly identical houses. What do we think will happen? Who will buy each house, and at what price? Will a land market assign each house to the person who values it most highly? What will be the welfare of each of the four people, two buyers and two sellers, involved in the transactions?

Suppose the two houses sell for the same price. In this case, the buyer of house 1,the house with the apple tree, is likely better off than his neighbor. He has the benefit of the apple tree without having had to pay more for it. This suggests that, in fact, his neighbor should have been prepared to pay a little more to buy house 1 and leave house 2 to the current occupant of house 1. This logic suggests that the houses should transact at prices that differ by exactly the value of the apple tree. If true, this means that the two households will end up indifferent between the house with the apple tree and the house without: the price difference completely reflects the value of the apple tree.

The basic logic of this location choice problem provides the foundation for much of urban economics. However, instead of the silly example of the apple tree, we would like to understand how land markets operate to reflect the value of proximity to central business districts, the cost of housing, the sorting of rich and poor into neighborhoods, or the puzzling fact that poor people are often unwilling to move short distances to neighborhoods where their children could expect much higher lifetime wages.

These are real, important problems, and each can be framed as a generalization of the problem of the apple tree above, where the 'apple tree' is replaced by differential proximity to economic opportunity or cost of housing. We would like to understand the extent to which land market prices reflect the value of these location specific attributes, and how their value is distributed between the different people who participate in the market.

In fact, the agenda we would like to investigate is much grander. We would like to understand how market forces, and their absence, affect the way the people organize where they live and work, that is, why cities are the way they are. We will shortly turn attention to these grander problems.

Most of the tools we have for thinking about these problems are based on one of two foundations. The first is the large class of 'moncentric city models'. In these models, a large number of identical households arrange themselves across a large number locations in a 'free mobility equilibrium', i.e., choices of land prices and locations such that no household wants to move. The second, which I will call economic geography or sorting models, is concerned with how households (and firms) that are not homogenous solve much the same problem.

The following three examples make precise the example of apple tree given above, and consider how land markets behave in three situations. These examples illustrate the way that land markets reflect the value of location specific attributes and divide their value among market participants in transparent and highly stylized examples. The intuition is general however. Being just a little reductive, we can build much of the monocentric city and economic geography literature from these two examples by generalizing the geography and making payoffs more realistic.

Example 1: Open city, homogenous households

The example described here illustrates the logic behind a large class of models that are collectively known as 'the monocentric city model' that will be the subject of much of the course. In particular, this example illustrates three ideas. First, it provides foundations for the 'free mobility equilibrium' that much of urban and regional economics relies upon. Second, it illustrates how location choice leads market price to 'capitalize', or reflect the value of, location specific attributes. Third, it demonstrates the useful relationship between welfare and land prices.

The example is based on an economy consisting of two households, two landlords and three locations. We will characterize Pure Strategy Nash Equilibrium choices of land rent and location choice.

There are three locations, $x \in \{0,1,2\}$. Locations 1 and 2 are the locations of interest, e.g., two particular parcels. Location 0 is an 'outside option'. The outside option is not congestible and can house both households, it is another city or neighborhood that lies outside our model. Locations 1 and 2 are congestible and can serve at most one household.

Landlords $j \in \{1,2\}$ each own own one location, and landlord j owns location j. Each landlord chooses the price at which they will offer to rent their parcel, $R_j \in \mathcal{R}_+$. Landlords' payoffs are $\pi_j = R_j$ if their parcel is occupied and 0 otherwise.

Index the two households by i. An action for a household is a choice of location, x_i, from among the three available. Each location affords an occupant a particular utility level, v_x, and occupying locations 1 or 2 requires a rent payment of R_j to the landlord. Payoffs for the households are given by

$$u_i = \begin{cases} v_0 & \text{if } x_i = 0 \\ v_1 - R_1 & \text{if } x_i = 1 \text{ and } x_{-i} \neq 1 \\ v_2 - R_2 & \text{if } x_i = 2 \text{ and } x_{-i} \neq 2 \\ -\infty & \text{if } x_i = x_{-i}\,, \end{cases} \tag{1}$$

where x_{-i} indicates the location choice of the other household.

In words, a household gets an exogenous level of utility at the outside option. It gets a parcel specific utility level net of a rental payment if it is the sole occupant of one of the two locations of interest. Finally, to prevent both households from choosing the same location, double occupancy results in disaster.

To begin, we assume that location 1 and 2 are attractive for some reason, so that $v_1 > v_0$ and $v_2 > v_0$. In the most common formulation of the monocentric city model, the value attached to parcels reflects the value of proximity to a place of work.

A strategy for a household is a choice of location for each possible choice of location by the other household and for each possible choice of price by the two landlords. That is, $s_i^x : \{0,1,2\} \times \mathcal{R}_+^2 \longrightarrow \{0,1,2\}$. A strategy for a landlord is a choice of price for each

Figure 1: Households' equilibrium location choice.

		Household 2		
		$x_2 = 0$	$x_2 = 1$	$x_2 = 2$
	$x_1 = 0$	(v_0,v_0)	$(v_0,v_1 - R_1)$	$(v_0,v_2 - R_2)$
Household 1	$x_1 = 1$	$(v_1 - R_1,v_0)$	$(-\infty, -\infty)$	$(v_1 - R_1,v_2 - R_2)$
	$x_1 = 2$	$(v_2 - R_2,v_0)$	$(v_2 - R_2,v_1 - R_1)$	$(-\infty, -\infty)$

choice of location by the two households and choice of price by the other landlord. That is $s_j^R : \{0,1,2\}^2 \times \mathcal{R}_+ \longrightarrow \mathcal{R}_+$. A strategy profile is a list of four such strategies, one for each landlord and household. A strategy profile is a Pure Strategy Nash Equilibria if no agent can make a unilateral deviation that results in an increase in her payoff.

Taking as given rent choices by the landlords, the decision faced by the two households can be described as the normal form game in figure 1.

Examining this game, it is clear that any Nash equilibrium outcome must have the following properties. First, location 1 and 2 are occupied by at most one household. Otherwise one of the double occupant households can deviate to the outside option for a higher payoff. Second, locations 1 and 2 can be occupied in equilibrium only if $v_i - R_i \geq v_0$. Otherwise, a household i can deviate to location 0 for a higher payoff. Third, if $v_i - R_i = v_0$, then households will be indifferent between location i and the outside option, and either choice is consistent with equilibria.

Summarizing in words, Nash strategies for the two households must be along the lines of the following. Taking R_1, R_2 as given, household 1 chooses whichever of locations 0,1,2 gives her the highest payoff. Household 2 is similar, but chooses between the outside option and unoccupied locations in $\{1,2\}$. In these strategies, household 1 'moves first' in that they do not avoid household 2's choice. The symmetric case where household 2 'moves first' would also be a Nash equilibrium, as would any variant in which the two have some way of coordinating to avoid double occupancy.

Given this, consider the landlords' choices. First, a landlord will never choose R_j such that $v_j - R_j < v_0$. In this case, both households prefer the outside option to location j, and the landlord's equilibrium payoff is zero. This choice of rent is dominated by any choice $R_j' \in (0,v_j - v_0)$, which results in positive rent in equilibrium. Second, a landlord will never choose R_j such that $v_j - R_j > v_0$. Such a strategy is dominated by one where $R_j' \in (R_j, v_j - v_0)$. That is, if landlord j chooses a price such that a household is strictly better off in location j than the outside option, then the landlord can raise her price a little without creating an incentive for the household to move.

The only remaining only possibility is that equilibrium land rent must satisfy $v_j - R_j^* = v_0$. That is, in equilibrium, it must be the case that landlords choose prices such that households are indifferent between a landlord's location and the outside option.

Suppose that both landlords choose R_j^*. In this case, the households are indifferent between locations 1 and 2 and the outside option. Suppose that one or more of the households chooses the outside option. In this case, the landlord's best response is to lower R_j slightly to induce a household to return, and so this cannot be a Nash equilibrium. It follows that the only candidate equilibria are those where landlords choose $R_j^* = v_j - v_0$

and households occupy locations 1 and 2. That is, in any equilibrium outcome we must have $R_j^* = v_j - v_0$ for both landlords, and the households occupy locations 1 and 2. It is easy to check that no unilateral deviation from this arrangement is profitable, given agents' strategies.

The analysis above is conducted under the assumption that locations 1 and 2 are intrinsically more attractive than the outside option. That is, $v_1 > v_0$ and $v_2 > v_0$. It is also possible that the locations are not attractive relative to outside option. In this case, households will only choose these locations if landlords subsidize them. Such a subsidy requires a landlord to incur negative profits when zero is possible. Thus, in the case where locations 1 or 2 are less attractive than the outside option, we should expect that they are unoccupied in all equilibria.

These equilibria have a number of interesting properties.

First, the equilibria are optimal in the sense that the highest value locations are occupied.

Second, in all equilibria in which locations 1 and 2 are occupied, households are indifferent between the outside option and the location where they end up. This occurs because the landlord captures all of the surplus associated with the availability of her parcel, $R_j^* = v_j - v_0$. This, in turn, has two important consequences.

In the context of this model, land rent is a direct measure of the surplus created by a location. If, for example, our set of two locations is a 'city', then the total land rent collected by landlord is a measure of the decrease in the sum of the total utility created by locations 1 and 2,$v_1 + v_2 - 2v_0$, that the economy would experience if these locations ceased to be available to households. There are few instance in economics where such a close link between an observable quantity and welfare can be drawn. With this said, we will see that that this link can break down as households become more heterogenous.

Second, much of the urban economics literature, including the monocentric city model, is based around models where a 'free mobility equilibrium' is assumed. Here, an equilibrium is assumed to occur if land rent is non-negative, all locations at least indifferent to the outside option are are occupied, and land rent adjusts so that all households are indifferent between all occupied locations and the outside option. In the linear city model we examine shortly, this is exactly how the equilibrium is defined. The example above demonstrates that the free mobility equilibrium can be rationalized by what is basically a Bertrand pricing game.

Somewhat reductively, one can think of much of the large literature on the monocentric model as generalizations of the example developed above, where much more thought is given to the values of v_i of particular parcels. For example, the extent to which value changes with distance from the center of a city or with the nature of housing production.

Example 2: Closed city, homogenous households

If we increase the outside option sufficiently, locations 1 and 2 must be unoccupied in equilibrium. Similarly, if we choose the v_i such that $v_1 > v_0 > v_2$, we must have only location 1 occupied in any equilibrium. In this sense, example 1 is an 'open city model'. That is, the outside option is set exogenously and the population of the 'city', here locations 1 and 2,is determined endogenously. A common alternative formulation is the 'closed city model'. In this version of the model, the population of the city is set exogenously and utility adjusts endogenously.

To see how such an equilibrium is different from the open city equilibrium we have just considered, consider this minor variant of the original model. Since migration out of the city is prohibited, restrict the set of locations to, $x \in \{1,2\}$, e.g., two particular parcels. There is a single household that is compelled to choose location 1 or 2. As before, both landlords choose rent and their payoffs remain the same as in the original. The household's payoff is similar to the original example, but adjusted to allow for the more restricted set of locations and smaller set of households,

$$u_i \;=\; \begin{cases} v_1 - R_1 & \text{if } x_i = 1 \\ v_2 - R_2 & \text{if } x_i = 2 \end{cases}, \tag{2}$$

Suppose, without loss of generality that location 1 is more desirable than location 2,$v_1 > v_2$. Strategies and strategy profiles are as before, adjusted for the smaller number of locations and households.

For given choices R_1, R_2, the household chooses location 1 if $v_1 - R_1 > v_2 - R_2$ and conversely. Trivially, the contrary choice creates an opportunity for a profitable unilateral deviation by the household. The household is indifferent between the two locations when $v_1 - R_1 = v_2 - R_2$.

First note that no landlord will rent at negative prices, $R_j = 0$ always gives a higher payoff. Given this, consider the landlord 1's problem. Suppose location 1 is occupied and $v_1 - R_1 > v_2 - R_2$. In this case, landlord 1 can increase R_1 slightly for a higher payoff. Thus, $v_1 - R_1 > v_2 - R_2$ is not an equilibrium. Now suppose that $v_1 - R_1 < v_2 - R_2$. In this case, landlord 1's payoff is zero, the household chooses location 2,and landlord 1 can attract the household and secure a positive payoff by choosing $R_1' < R_1$ such that $v_1 - R_1' > v_2 - R_2$. Since is $v_1 > v_2$ is assumed, such an R_1' must exist. The only remaining possibilities are that $v_1 - R_1 = v_2 - R_2$ in equilibrium.

Suppose that $v_1 - R_1 = v_2 - R_2$ and $R_2 > 0$. Then whichever landlord owns the unoccupied location can lower her price to attract the household. Since $R_j \geq 0$, this leaves only $R_1 = v_1 - v_2, R_2 = 0$. At these prices, the household is indifferent between the two locations. Suppose the household chooses location 2. In this case, landlord 1 can deviate to $R_1' \in (0, v_1 - v_2)$ to attract the household. Thus, this cannot be an equilibrium. Therefore, the only possible equilibrium occurs when $R_1 = v_1 - v_2, R_2 = 0$ and the household chooses location 1. It is straightforward to see that neither landlord nor household has a profitable unilateral deviation, taking opposing strategies as given.

Comparing the equilibrium of the closed city game to the open city game, we see the fundamental difference between the two. In the open city game, the outside option is set exogenously. In the closed city, the role of 'outside option' is played by the most attractive unoccupied location, here location 2. That is, in the open city game, prices reflect the difference between household values for the outside option and each occupied location. In the closed city game, prices reflect the difference in household values between each occupied location and the most attractive unoccupied location.

Example 3: Open city, heterogenous agents

The examples above capture the central logic of a large class of monocentric city models. When landlords and households behave as selfish maximizers and when households can move costlessly between locations, then equilibrium should have the following two properties. First, households will be indifferent between any occupied location and the

Figure 2: Households' equilibrium location choice.

		Household 2		
		$x_2 = 0$	$x_2 = 1$	$x_2 = 2$
	$x_1 = 0$	(v_0^1, v_0^2)	$(v_0^1, v - R_1)$	$(v_0^1, v - R_2)$
Household 1	$x_1 = 1$	$(v - R_1, v_0^2)$	$(-\infty, -\infty)$	$(v - R_1, v - R_2)$
	$x_1 = 2$	$(v - R_2, v_0^2)$	$(v - R_2, v - R_1)$	$(-\infty, -\infty)$

best unoccupied location. Second, land rents will reflect the difference in location specific utility between each location and the best unoccupied location. In this sense, land rent capitalizes location specific attributes. In the open city model, this has an important further implication, aggregate land rent equals the sum of what households would pay to occupy the subject city rather than the outside option. In this sense, aggregate land rent tells us something important about welfare.

In the examples above, the households were assumed to have identical preferences over locations. As we introduce heterogeneity in household preferences over locations, we open the door to the possibility that the price of a location is driven by a household with preferences different from those of the household that occupies the location in equilibrium. When this occurs, it may happen that land rent does not fully capitalize parcel specific values. Hence, welfare calculations need to be concerned with consumer surplus in addition to land rent. These issues rarely arise in the class of moncentric city models, but are central to models of sorting, and in particular, to the recent literature in economic geography. The example below illustrates.

The example is based on an economy consisting of two households, two landlords and three locations. Notation is as before and we will characterize Pure Strategy Nash Equilibrium choices of land rent and location choice. The example differs from those above in that households have different values for the outside option, but identical values for the the two parcels in the city.

More specifically, actions and payoffs for landlords are unchanged and payoffs for the households are given by

$$u_i = \begin{cases} v_0^i & \text{if } x_i = 0 \\ v - R_1 & \text{if } x_i = 1 \text{ and } x_{-i} \neq 1 \\ v - R_2 & \text{if } x_i = 2 \text{ and } x_{-i} \neq 2 \\ -\infty & \text{if } x_i = x_{-i}\,, \end{cases} \tag{3}$$

where x_{-i} indicates the location choice of the other household.

In words, a household gets a household specific reservation level of utility at the outside option. It gets a city specific utility level, net of a rental payment if it is the sole occupant of one of the two locations of interest. As before, to prevent both households from choosing the same location, double occupancy results in disaster. Assume that location 1 and 2 are attractive for some reason, so that $v > v_0^i$ for both households. Finally, without loss of generality, assume $v_0^1 < v_0^2$. That is, household 1's outside option is worse than that of household 2.

Taking as given rent choices by the landlords, the decision faced by the two households can be described as the normal form game in figure 2.

Examining this game, it is clear that any Nash equilibrium outcome must have the following properties. First, location 1 and 2 are occupied by at most one household. Otherwise one of the double occupant households can deviate to the outside option for a higher payoff. Second, locations 1 and 2 can be occupied in equilibrium only if $v - R_j \geq v_0^i$ for some household i. Otherwise, a household i can deviate to location 0 for a higher payoff. Third, if $v - R_j = v_0^i$, then household i will be indifferent between location j and the outside option, and either choice is consistent with equilibria.

Summarizing in words, Nash strategies for the two households must be along the lines of the following. Taking R_1, R_2 as given, household 1 chooses between locations 0,1,2 according to whichever gives her the highest payoff. Household 2 is similar, but chooses between the outside option and unoccupied locations in {1,2}. In these strategies, household 1 'moves first' in that they do not avoid household 2's choice. The symmetric case where household 2 'moves first' would also be Nash, as would any variant in which the two have some way of coordinating to avoid double occupancy.

Understanding landlord strategies in this game is slightly more subtle than in the game with homogenous agents. To begin, note that a landlord is always better off choosing a non-negative rent than a negative one, even if it means her parcel is vacant. In addition, a landlord choosing $R^j > v - v_0^1$ is assured of not being occupied, and so this rent is inferior to a small positive R^j which must attract a household. In between these two values, it is a little more difficult to see what happens.

To begin, consider figure 3. The x-axis in this figure indicates landlord 1's choice of rent, the y-axis landlord 2's choice. As discussed above, the interesting region is $R_j \in [0, v - v_0^1]$. This is the region illustrated in the figure. Suppose that landlord 1 chooses $0 \leq R_1 < v - v_0^2$. In this case, both households are strictly better off in location 1 than in the outside options. Given household strategies, there can be no equilibrium in which location 1 is unoccupied.

But if $R_j \in [0, v - v_0^1)$ then landlord j can deviate to a slightly higher rent without dislodging her tenant. It follows that there can be no Nash equilibrium in which either landlord chooses $R_j \in [0, v - v_0^2)$. Thus, no rent pairs in the dark gray box can be a Nash equilibrium outcome.

Next suppose that both landlords choose $R_j \in (v - v_0^2, v - v_0^1]$. That is, high enough that that household 2 prefers the outside option, but not so high that household 1 does so. These are the rent pairs that lie in the light gray box of figure 3. This also cannot be a Nash equilibrium. One of the landlords is certain to be unoccupied and can improve her payoff by choosing R_j low enough to attract household 2 away from the outside option.

Similar logic eliminates the unshaded rectangles that share a side with the dark gray and light gray boxes. The possibilities that remain lie on the locus ADC.

On the locus ADC, one landlord chooses $R_j = v - v_0^2$ and the other chooses $R_{-j} \in [v - v_0^2, v - v_0^1]$. Suppose it is landlord 1 who chooses $R_1 = v - v_0^2$. In this case, household 2 is indifferent between location 1 and her outside option, and household 1 strictly prefers location 1 to her outside option. Suppose household 2 occupies location 1 and location 2 is empty. This is not a Nash equilibrium since landlord 1 has an incentive to reduce her price to attract the household at the outside option. Similarly if neither location is occupied, both landlords can deviate to a lower price to attract a household and increase their payoffs.

Finally, suppose that both locations are occupied. The only way that this could occur is if household 2 occupies location 1 (and is just indifferent between location 1 and her outside option), while household 1 occupies location 2 (which she at least weakly prefers to her outside option). This also is not a Nash equilibrium. To see this, note that landlord 1 can increase her rent to something between $(v - v_0^2, R_2)$. In this case, household 2 no longer prefers location 1 to her outside options, and household 1 prefers location 1 to the more highly priced location 2. Thus the only remaining possibility is when both landlords choose $R_j = v - v_0^2$ and households occupy locations 1 and 2. That is, point D in figure 3.

This is, in fact, a Nash equilibrium outcome under these strategies. To see this, note that neither household has an incentive to deviate. Household 2 is indifferent between location 1 or 2 and her outside option. Household 1 strictly prefers location 1 or 2 to her outside option. The landlord housing household 2 has no incentive to deviate. Increasing her rent will lead to a vacant parcel, decreasing rent leads to an immediate decrease in payout. What about the landlord housing household 1? If this landlord increases her price slightly, household 1 will not deviate to the outside option. However, the other landlord's best response to such a price increase is to increase her own price by slightly less, displacing household 2 and stealing away household 1. Thus, this outcome is a Nash equilibrium.

This logic is somewhat complicated, but is important for three reasons. First, the large class of urban and economic geography models that relies on heterogeneous agents can be thought as generalizations of this simple model in which much more attention is directed to describing the sources of preference heterogeneiety and of location value. Thus, understanding this simple model provides insight into how widely used and much more complicated models work. Second, this example makes explicit that equilibrium can result from a simple pricing game. In contrast, much of the literature simply defines an equilibrium to occur when the marginal agent is indifferent between their location and their outside option.

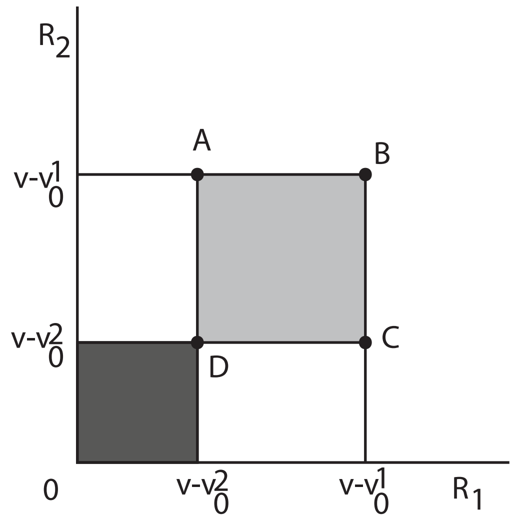
Finally, this example shows how the relationship between land rent and welfare starts to break down with preference heterogeneity. In particular, since both households pay the rent that makes household 2 indifferent between locations 1 and 2 and the outside option, household 1 receives a payoff $v - (v - v_0^2) = v_0^2$. This is strictly greater than her outside option, v_0^1. Thus, with heterogenous outside options and free mobility, a competitive land market fails to completely capitalize the value of the city. Household 1,the household with the worse outside option, captures some of the benefit and is stricly better off than she would be in the absence of the city.

Therefore, in examples where outside options are heterogenous, land rent does not provide the same information about welfare as it does in a world with homogenous outside options. In order to consider welfare, we must also consider consumers surplus. In the context of this example, this is simply $v_0^2 - v_0^1$.

Summary

When agents have similar preferences, land markets completely capitalize the difference in value of location specific attributes for between any occupied parcel and the best unoccupied parcel. This has three important implications. First it means that land rent provides a rare, easy to observe opportunity to learn about the value that people assign to different location specific goods. Second, the entire value of these location specific attributes is

Figure 3: Landlords' choices of rent with heterogenous agents



'capitalized into land prices' and ends up in landowners' pockets. Households end up indifferent between all locations. These basic results hold if households have the same preferences over location specific attributes.

Whether the city is open or closed is important in that it endogenizes the choice of the best unoccupied location, the reference location that households use to evaluate prices of other locations. This changes the level of land prices, but does not change the fact that differences between location prices reflect difference in the value of location specific attributes.

Most applications of the monocentric city model are concerned with agents whose preferences are similar, and who choose across a large set of potential locations. These models typically result in full capitalization so that landlords capture the full value of location specific attributes.

Preference heterogeneity can, but does not always, cause this logic to break down. In the economic goegraphy and sorting literature, preference heterogeneity typically plays an important role. In addition to observing land rent, welfare statements require a guess at the distribution of household preferences. Indeed, estimating such preference heterogeneity is typically a central part of using these tools.