CHAPTER SIX

Monocentric Cities

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6.1 Introduction

Anyone who is even a casual student of cities has noted that, within a particular city, the economic landscape can vary dramatically, especially with proximity to the central business district. The cost of renting an apartment in the city proper can be hundreds of dollars per month higher than for a comparable apartment in an outlying suburb. And the choice might mean a difference between living in a high-rise apartment building (city proper) and a low-rise townhouse development (outlying suburb). The monocentric city model is a descriptive model of resource allocation in a city that was designed to explain precisely such phenomena. Its basic development occurred in the 1960s and 1970s, largely through the work of William Alonso, Richard Muth, and Edwin Mills (Alonso 1964; Muth 1969; Mills 1972). Since that time, cities have become increasingly polycentric, and the monocentric city model, with its assumption of a single concentration of employment, has been criticized on the grounds that the cities it explains are from a different era. My response to this criticism is twofold. First, there are many urban areas for which the assumption of a single employment center serves as a reasonable approximation. But more importantly, the economic forces that arise in monocentric cities are crucial to understanding polycentric cities, and this makes the simpler monocentric city model the natural place to start.

The objective of this essay is to take a monocentric city model that strikes a balance between richness and simplicity and set it out so that it is accessible to a student with no training in economic theory beyond an undergraduate course in micro theory. The chapter relies as much as possible on diagrammatic analysis and makes only minimal use of mathematics. The specific model we set out is taken from Brueckner (1987), to which the reader is referred for a more advanced treatment of the monocentric city model. We are indebted to that paper not only for its model, but also for the insights it provides on the model's operation.

Section 6.2 sets out the model and shows how it can be used to analyze the internal structure of a city. Section 6.3 illustrates how the model can be used to make comparisons across cities. Section 6.4 concludes with a description of some of the interesting ways in which the model has been extended.

6.2 A Model of a City

Imagine a circular city in a featureless plain. All employment is in a central business district (CBD), which we take to be a point at the city's center. The city is populated by N identical individuals. Each individual makes one round trip per day to the CBD, where he works a fixed number of hours and receives a daily wage of y. The greater the radial distance from the center at which an individual resides, the greater is the cost of this commute. Specifically, for an individual who lives at radial distance x, daily commuting cost is assumed to be tx, where $t > 0$ is a constant round-trip commuting cost per unit distance. The cost of commuting will be treated as an out-of-pocket cost, rather than a time cost.

An individual has a utility function $v(q,c)$, where q is his consumption of the services of housing, c represents his consumption of a composite of other goods, and both have a positive effect on utility. In actuality, the services that individuals derive from their residences are multidimensional, floor area and yard space being just a couple; here, these various attributes are represented by the scalar variable q, which we will simply refer to as consumption of housing.

An individual who lives at radial distance x faces a rental price (per unit) of housing of $p(x)$. We will refer to this simply as the price of housing.

As for the composite commodity, it can either be produced in the CBD or imported. Its price is assumed to be spatially invariant and is set at unity.

An individual's objective is to maximize $v(q,c)$. In doing so, he is subject to the budget constraint

$$p(x)q + c = y - tx. \tag{6.1}$$

There are two aspects to the problem. One is the individual's location choice, x. The second is the choice of a housing consumption q and other goods consumption c at the chosen location, and is depicted in Figure 6.1 for an individual whose location choice is denoted by x_0. The c-intercept of the budget line is given by $y - tx_0$, which is income net of commuting cost. For convenience, we will refer to this as net income. The slope of the budget line is simply $-p(x_0)$. The label $p(x_0)$ refers to its absolute value. Equilibrium occurs at e_0, a point at which one of the individual's indifference curves (assumed to be strictly convex) is tangent to the budget line. The equilibrium consumption of housing at this location is q_0.

The fact that individuals have a location choice together with the assumption that they are identical means that the same utility level must be realized at all residential locations. The reason is simply that any one individual can replicate the location and consumption decisions of anyone else. The mechanism for satisfying the equal-utility condition is spatial variation in the price of housing. In order to offset the reduction in net income associated with an increase in x (because of higher commuting cost), the price of housing must decrease with x.

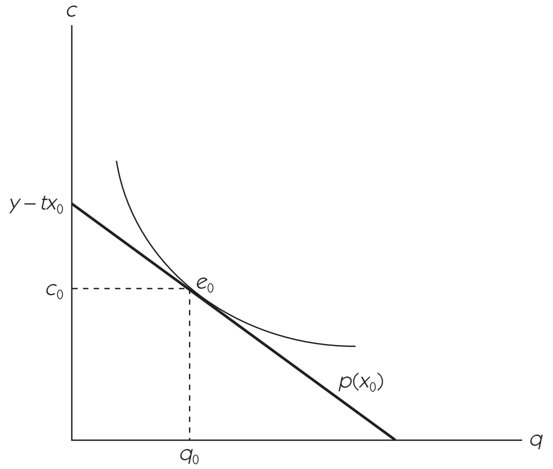

Figure 6.1 Equilibrium housing consumption at a particular location.

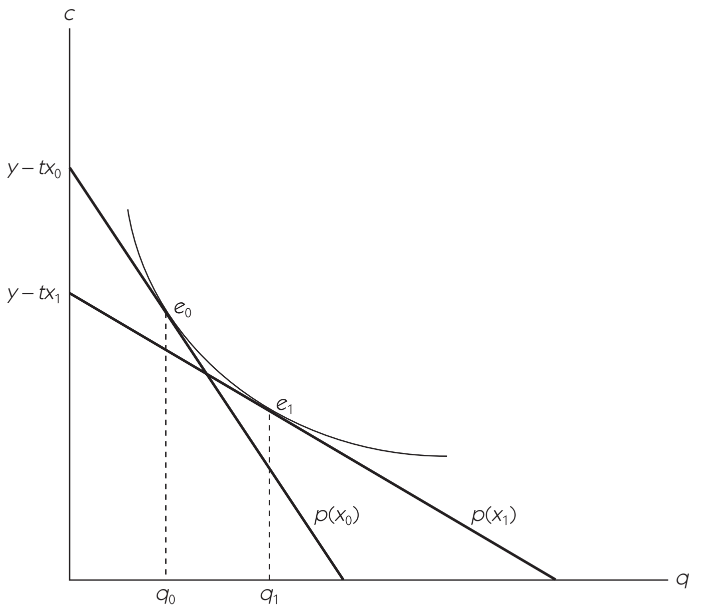

Figure 6.2 The price of housing varies spatially to achieve locational equilibrium.

This is shown in Figure 6.2, in which the indifference curve drawn is assumed to correspond to the common utility level in equilibrium, and x_1 and x_0 represent any two locations such that $x_1 > x_0$. $p(x_0)$ and $p(x_1)$ are such that the equilibrium utility level is just attainable with budget lines whose respective c-intercepts are

$y - tx_0$ and $y - tx_1$. Net income is lower at x_1 than at x_0, meaning that the budget line there must be flatter. Thus, $p(x_1) < p(x_0)$. We therefore have the following:

Property 1. The rental price of housing decreases with distance from the CBD.

Figure 6.2 can also be used to compare the housing consumption of individuals at different locations. Since the budget line is flatter at x_1 than at x_0, the equilibrium indifference curve is flatter at e_1 than at e_0. With strictly convex indifference curves, this means that the position of e_1 relative to e_0 on the equilibrium indifference curve is to the southeast, or that $q_1 > q_0$. We state this as follows:

Property 2. Individuals who live further from the CBD have higher consumption levels of housing.

To help set the stage for what will come later, we next introduce the notion of a *bid-rent curve*. In Figure 6.2, $p(x_0)$ is the highest price that an individual who lives at x_0 can pay for housing, while still attaining the level of utility associated with the indifference curve in the figure. If the price of housing at x_0 were any higher, the budget line there would be steeper, and the indifference curve in question would be unattainable. $p(x_0)$ is referred to as a bid rent for housing. The preceding discussion of equilibrium housing prices makes clear that for a given level of utility, the bid rent for housing is a decreasing function of x. The bid rent for housing is also affected by a change in the level of utility. This can be seen from Figure 6.3, which compares the bid rent for housing for two different utility levels at the same location. The fact that location is held fixed is reflected in both budget lines having the same c-intercept. The higher indifference curve can only be achieved with a flatter budget line, which is to say that the bid rent for housing must be lower. Denoting the bid rent for housing by $\tilde{p}$ and the level of utility by u, we can write $\tilde{p} = \tilde{p}(x,u)$, where $\tilde{p}$ is a decreasing function of both x and u. The graph of $\tilde{p}$ versus x for a given value of u is called a bid-rent curve for housing. Figure 6.4 shows two members of a family of bid-rent curves. Since $\tilde{p}$ is a decreasing function of u, utility is higher along the lower of the curves. One of the bid-rent curves corresponds to the equilibrium utility level. That bid-rent curve is the graph of equilibrium housing prices.

The quantity of housing demanded by an individual who faces his bid rent is denoted by $\tilde{q}(x,u)$. $\tilde{q}$ is an increasing function of x which, with u set at its equilibrium level, is the equilibrium $q(x)$ function. Under the additional assumption that housing is a normal good (an individual's housing demand positively related to his income), $\tilde{q}$ is also an increasing function of u. In Figure 6.3, the difference between q'' and q' is partly due to an income effect. The normality assumption insures that $q'' > q'$.

For the remainder of the chapter, housing will be assumed to be a normal good. The assumption is weak on prior grounds and is strongly supported empirically.

We now turn to the behavior of housing producers. Housing producers are assumed to be identical and to maximize profit, taking prices as given. An individual housing producer can produce housing at any or all x. Its output of

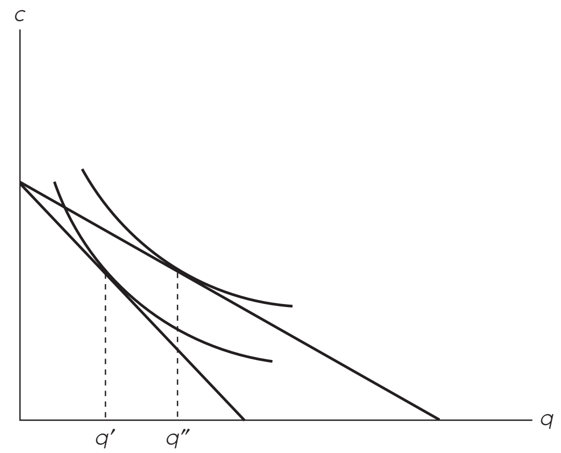

Figure 6.3 A higher level of utility decreases the bid rent for housing.

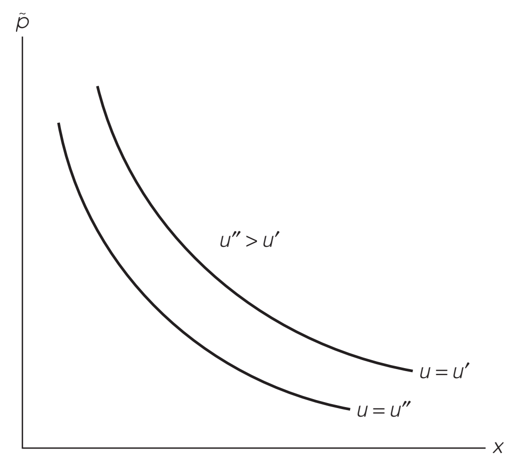

Figure 6.4 A family of bid-rent curves for housing.

housing at a location, $H(x)$, is given by the constant returns to scale (CRS) production function $H(x) = H(L(x),K(x))$, where $K(x)$ is its employment of capital at x and $L(x)$ is its employment of land at x. The rental price of land at x is $r(x)$, while capital has a spatially invariant rental price of i. i is exogenously determined in a national capital market; the rental price of land is endogenous. At a particular location x, a housing producer generates a profit given by $p(x)H(L(x), K(x)) - r(x)L(x) - iK(x)$. The problem it faces is to maximize this with respect to $L(x)$ and $K(x)$ for all x.

To see what the problem entails, we begin by recalling some classic results on constant returns to scale and applying them to our CRS housing producers. The first is that a CRS firm that is a price-taker in factor markets has a long-run average cost that is independent of its output. This means that an individual housing producer has a long-run average cost curve at a location that is horizontal, which we represent by AC in Figure 6.5. Second, under the additional assumption that firms act as price-takers in the product market, the long-run supply curve for an individual firm, and hence for the market as a whole, becomes perfectly elastic at a price equal to the constant level of long-run average cost. In Figure 6.5, AC is therefore the long-run market supply curve for housing at x, meaning that for equilibrium in the housing market at a location, the price of housing and the average cost of housing must be equal. We write this as

$$p(x) = AC(r(x),i), \tag{6.2}$$

since in Figure 6.5, an increase in either $r(x)$ or i leads to an upward shift in AC. As x increases, $p(x)$ decreases (Property 1). Together with equation (6.2), this implies that the same must be true of the average cost of housing production. Since the rental price of capital is spatially invariant, this can come about only if $r(x)$ is a decreasing function of x. We state this important property as follows:

Property 3. The rental price of land decreases with distance from the CBD.

In real-world urban areas, the form that housing takes often varies dramatically with distance from the city center. At the closest-in locations, the dominant form of housing might be high-rise apartment buildings or condominiums. In less central areas of the inner city, housing might be predominantly three- to five-story apartment buildings. Out in the suburbs, this might give way to single-family homes, with a general tendency for lot sizes to become progressively larger in the more outlying suburbs. We can summarize this by saying that as distance from the city center increases, there is in general a decrease in the capital–land ratio in housing production. We will now proceed to demonstrate that this is exactly what our model predicts.

The result can be seen in terms of Figure 6.6, in which we have sketched the so-called unit isoquant – those combinations of labor and capital from which an individual housing producer can produce exactly one unit of housing. As shown in the diagram, isoquants are assumed to be strictly convex. To begin with, consider some particular location x_0, and a housing producer that chooses to operate on the unit isoquant there. In order to maximize its profit, the firm must employ the cost-minimizing input combination. This is the point e_0, at which an isocost line that reflects factor prices at x_0 is tangent to the unit isoquant. Factor prices are reflected in the slope of the isocost line, which is $-\,r(x_0)/i$. Now consider the capital–land ratio at e_0. This is commonly referred to as structural density and is given by the slope of a ray from the origin through e_0. The next thing to see is that all firms that produce housing at x_0 have the same structural density, regardless of the quantity of housing that they produce. This is another

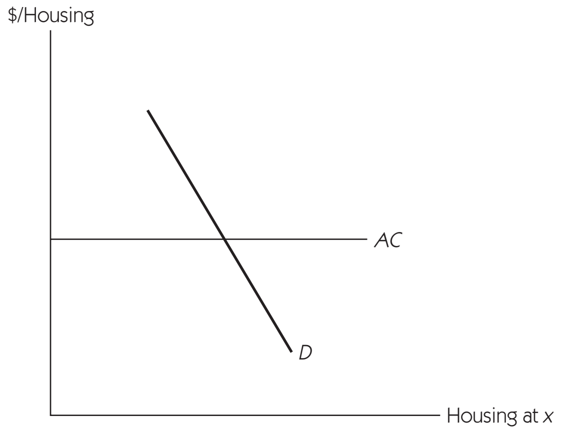

Figure 6.5 Equilibrium in the housing market.

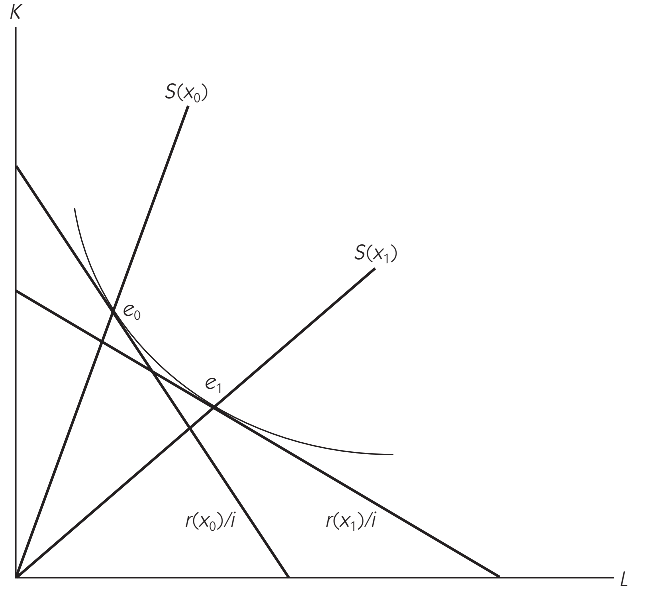

Figure 6.6 Structural density decreases with distance from the CBD.

implication of constant returns to scale. It comes from the fact that for a given ray from the origin in Figure 6.6, the isoquants of a CRS production function are identically sloped at all points along the ray. Thus, if location and hence factor prices are held fixed, all points of tangency between isoquants and isocost lines involve the same structural density. In what follows, the structural density at x is denoted by $S(x)$.

Now consider some second location, $x_1 > x_0$. From Property 3, $r(x_1) < r(x_0)$, meaning that isocost lines are flatter at x_1. Together with strictly convex isoquants, this means that the cost-minimizing point on the unit isoquant, the point e_1, lies on a flatter ray from the origin than does e_0. We therefore have $S(x_1) < S(x_0)$. We state this as follows:

Property 4. Structural density decreases with distance from the CBD.

By way of additional stage-setting, we next consider the notion of a bid-rent curve for land. It has a strong connection to the previously considered notion of a bid-rent curve for housing. Consider some particular location x, and ask "If the price at which a housing producer can sell housing at x is the bid rent for housing $\tilde{p}(x,u)$, then what is the highest price it can pay for land at x, while still satisfying the zero-profit condition (6.2)?" This maximum bid is called a bid rent for land. Denoting it by $\tilde{r}(x,u)$, we have

$$\tilde{p}(x,u) = AC(\tilde{r}(x,u),i). \tag{6.3}$$

We have previously seen that an increase in either x or u decreases $\tilde{p}$. Together with equation (6.3), this implies that an increase in x or u also decreases $\tilde{r}$. The graph of $\tilde{r}$ versus x for a given value of u is called a bid-rent curve for land. One member of the family of bid-rent curves for land corresponds to the equilibrium utility level. That bid-rent curve is the graph of equilibrium land prices.

The structural density chosen by a housing producer that faces its bid rent for land is denoted by $\tilde{S}(x,u)$. $\tilde{S}$ is a decreasing function of x which, with u set at its equilibrium level, is the equilibrium $S(x)$ function. An increase in u also decreases $\tilde{S}$.

Another feature of cities that our model explains is the tendency for net residential density – population per unit of land used for residential purposes – to decline with distance from the CBD. Denoting net residential density at x by $D(x)$, the simplest way to see this is to write $D(x) = h(x)/q(x)$, where $h(x)$ is housing output per unit of land allocated to housing production at x. An increase in x increases $q(x)$ (Property 2), so it is enough to show that $h(x)$ decreases. This is nothing more than the fact that in Figure 6.6, housing output is the same at e_1 as at e_0, but is produced with a greater input of land (given CRS, it is enough to consider a movement along the unit isoquant). We can therefore state the following:

Property 5. Net residential density decreases with distance from the CBD.

To complete our stage-setting, as we have called it, suppose that all economic agents face their bid rents – for housing in the case of individuals and land in the case of housing producers – and consider the resulting net residential density, which we will denote by $\tilde{D}(x,u)$. $\tilde{D}$ is a decreasing function of x which, with u set at its equilibrium level, is the equilibrium $D(x)$ function. An increase in u also decreases $\tilde{D}$.

Through our so-called stage-setting, we have seen how the model gives rise to the functions $\tilde{p}(x,u)$, $\tilde{q}(x,u)$, $\tilde{r}(x,u)$, $\tilde{S}(x,u)$, and $\tilde{D}(x,u)$. If we had a way of determining u, then we could use these functions to determine equilibrium housing and land prices at all locations in the city, as well as other equilibrium spatial profiles. We would also be able to determine the city's equilibrium size. This is illustrated in Figure 6.7, in which $\bar{x}$ is the city's radius and r_A is a spatially invariant bid rent for land on the part of agriculture. To the left of $\bar{x}$ in the diagram, the bid rent for land on the part of agriculture is less than for housing production, so that all land up for bid is secured by housing producers. To the right of $\bar{x}$, it is the agricultural bid rent that is higher, so that only agriculture secures land. Since no housing is produced outside of $\bar{x}$, that is the city's equilibrium radius.

As shown in Figure 6.7, $\bar{x}$ occurs where the bid-rent curves for land in its alternative uses in housing and agriculture intersect. We can express this through the equation

$$\tilde{r}(\bar{x},u) = r_A, \tag{6.4}$$

which can be viewed as an equation for determining $\bar{x}$ in terms of u.

Exactly where u comes from depends on whether the city is "open" or "closed." In the case of an open city, the value of u is exogenous. It is the prevailing level of utility elsewhere in the economy, and is achieved in the city through costless migration. The city's population, N, is therefore endogenous and is determined as follows. At each radial distance x, assume that a fraction $\theta/2\pi$ of the land at x is available for the endogenous uses housing and agriculture. Then the number of individuals that can be housed in a ring of infinitesimal width dx at radial distance x is $\tilde{D}(x,u)\theta x \, \mathrm{d}x$. The city's equilibrium population is the number of individuals that can be housed inside $\bar{x}$, or

$$N = \int_0^{\bar{x}} \tilde{D}(x,u)\theta x \, \mathrm{d}x. \tag{6.5}$$

In the case of a closed city, the size of the city's population is exogenous, and the equilibrium utility level is endogenous. Equation (6.5) must hold, but now with N given. Like equation (6.4), it involves the endogenous variables $\bar{x}$ and u. $\bar{x}$ and u can therefore be determined from equations (6.4) and (6.5). Once this solution is obtained, all equilibrium spatial profiles can be determined.

6.3 Comparative Statics

One of the most interesting features of an economic model is its comparative-statics properties. In general, these have to do with how a model's parameters affect its solution. For the model of the preceding section, a very thorough comparative-statics analysis can be found in Brueckner (1987). The objective here is only to demonstrate how the analysis is carried out. For this purpose, we take the closed city variant of the model and work through the effect of an increase in

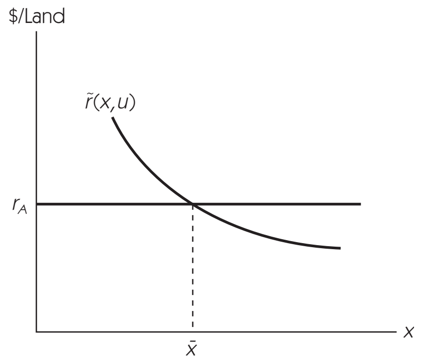

Figure 6.7 Equality of bid rents at the boundary.

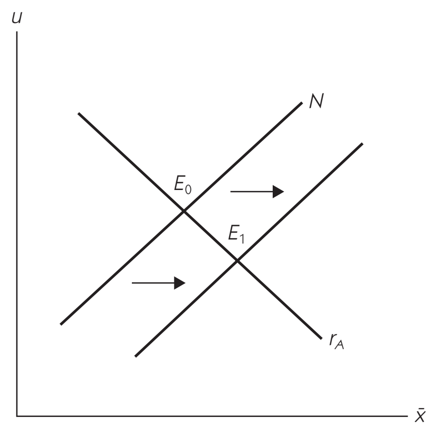

Figure 6.8 Population increase for a closed city.

N. This amounts to asking, "If two cities are identical in all respects except for population, how would they be expected to differ in terms of housing and land prices, geographical size, and so on?"

One of the keys to the analysis is our knowledge of the effect of an increase in u in the functions $\tilde{p}(x,u)$, $\tilde{q}(x,u)$, $\tilde{r}(x,u)$, $\tilde{S}(x,u)$, and $\tilde{D}(x,u)$. Collecting results from the preceding section, we have that $\tilde{p}$, $\tilde{r}$, $\tilde{S}$, and $\tilde{D}$ all decrease with u, while $\tilde{q}$ is increasing.

Given an initial value of N, a solution to equations (6.4) and (6.5) for $\bar{x}$ and u can be represented by a point such as E_0 in Figure 6.8. The curve labeled r_A represents those combinations of values for $\bar{x}$ and u for which equation (6.4) holds, while the curve labeled N represents the same for equation (6.5). As shown in the figure, the "r_A-curve" slopes downward, while the "N-curve" slopes upward. First consider the r_A-curve. Its downward slope can be seen from Figure 6.7, which illustrates a solution to equation (6.4). Since $\tilde{r}$ is a decreasing function of

u, an increase in u results in a downward shift in the bid-rent curve for land in housing production, and its intersection with the agricultural bid-rent curve occurs at a lower value of $\bar{x}$. Now consider the N-curve. Since $\tilde{D}$ is a decreasing function of u, an increase in u for which $\bar{x}$ is held fixed decreases the right-hand side of equation (6.5) (a reduction in the number of individuals who are housed). Thus, the only way to house a given number of individuals while providing them with a higher level of utility is to increase $\bar{x}$.

The stage is now set to analyze an increase in N. The first thing to note is that, in the system consisting of equations (6.4) and (6.5), N is a shift parameter of the latter, but not the former. In terms of Figure 6.8, this means that only the N-curve will shift. We can easily work out the direction of the shift. Given an increase in N, the same increase must occur in the right-hand side of equation (6.5) (the number of individuals actually housed). For any given value of u, the only way this can happen is with an increase in $\bar{x}$. Thus, when population increases, the N-curve shifts rightward. The new equilibrium ($\bar{x}$,u) combination is E_1 in Figure 6.8. Thus, an increase in N increases $\bar{x}$, while decreasing u.

Other results can now be easily obtained. The fact that $\tilde{p}$ is a decreasing function of u, together with the decline in the equilibrium utility level, means that there is an increase in $\tilde{p}$ for all x. This implies an increase in the rental price of housing at all locations out to the base city's outer edge. By an essentially identical analysis, the same is true of the rental price of land and both structural and net residential density. At any location x, a resident of the larger city consumes a smaller quantity of housing.

Of the preceding results, the only one that might seem surprising is that individuals who live in the larger city have a lower utility level. This is not necessarily realistic, in part because we have made no account of the possibility that agglomeration economies result in the larger city having a higher wage rate (see, e.g., Henderson 1985). But for what it is, the result can be seen as follows. Since utility is equal at all locations in each city, it is enough to compare the utility of Ms A, a resident of the smaller city who lives at its outer edge, with that of Mr B, who lives at the outer edge of the larger city. Because these individuals live at the outer edges of their respective cities, the price they pay for the housing they consume is the same (the rental price of land at the periphery is the same in both cities and is r_A). Thus, the individual who achieves the higher utility level is the one for whom income net of commuting cost is greater. This is clearly Ms A, since between the two, she has the shorter commute.

If an exogenous change in t or y occurs, things are not as simple. A particular value of each of these parameters is subsumed into the functions $\tilde{p}(x,u)$ and $\tilde{q}(x,u)$, and therefore $\tilde{r}(x,u)$, $\tilde{S}(x,u)$, and $\tilde{D}(x,u)$. As before, the first step is to determine how the parameter change affects the solution to equations (6.4) and (6.5) for $\bar{x}$ and u. But in order to determine the effect on, say, structural density at a location, one has to take into account both the direct effect of the parameter change and the indirect effect of the utility change that it induces. An exogenous change in population size involves only the second of these.

6.4 EXTENSIONS

The monocentric city model has been extended to include a variety of interesting features, each enhancing the model's realism. We conclude by describing some of the interesting directions in which the model has been taken:

- *Different income groups*. A number of authors, including Muth (1969), have generalized the model to allow for different income groups. An interesting new question arises, namely the following: In relation to individuals with low incomes, do individuals with high incomes live closer or farther from the CBD? Although the correct answer to this question is "It depends," usually the answer is "farther." The reason is that, *ceteris paribus*, higher-income individuals consume more housing (all it takes for this is for housing to be a normal good), giving them an added incentive to locate where the rental price of housing is lower, which is further from the CBD. A good source of information for the comparative statics of such a model is Hartwick, Schweizer, and Varaiya (1976).
- *Traffic congestion*. Another way in which the model has been extended is to take into account that commuting entails a very important time cost, and that traffic congestion makes commuting time from a location endogenous (see, e.g., Solow 1972). At any radial distance x, the time it takes to travel a unit distance is assumed to be an increasing function of the number of individuals who live between x and $\bar{x}$, since these outliers of x constitute the traffic at x. This naturally raises questions about how commuting should be priced and how land should be allocated to congestion-reducing road capacity, and has been a primary focus of work in this area.
- *Modal choice and nonradial access costs*. In the model of section 6.2, an individual's commuting cost depends only on the radial distance from his residence to the CBD. In order for this to be plausible, the city's radial road system must be sufficiently dense that circumferential access costs can safely be ignored. Anas and Moses (1979) have analyzed an extension of the monocentric city model in which radial roads are discretely spaced and entail costly circumferential access. Consistent with what is commonly observed in cities, the city will tend to develop along its radial roads, and unlike in the basic monocentric city model, the rural/urban boundary will be noncircular. The Anas and Moses paper is also one of several to have introduced mode choice into the model.
- *The fully closed city*. The model of section 6.2 implicitly assumes that the land employed by housing producers is rented from landowners who are not residents of the city. This follows from the fact that the income of residents is assumed to be exogenous, while aggregate land rents are endogenous. In a variant of the model known as the "fully closed city," aggregate land rents are a source of income to residents. One way to tell the story is that the land employed by housing producers is rented from the city, which in turn rents it

from either the federal government or from farmers, doing so at a rental price equal to the agricultural bid rent r_A. Since the rental price paid by housing producers exceeds r_A, this process generates a surplus for the city, which Arnott and Stiglitz (1981) termed differential land rents. In a fully closed city with identical consumers, each consumer receives an equal share of differential land rents as a transfer from the city. The fact that income is endogenous makes it the most complicated version of the model. For more on the fully closed city model, see Pines and Sadka (1986).

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