EC1410 – Spring 2025

Midterm Solutions

8:40-9:40am, March 11, 2026

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1. (5) In the context of this class, what does the acronym 'MSA' stand for?

   Metropolitan Statistical Area

2. (a) Assume we have the setup of the monocentric city model with housing, as in the lecture. Assume as well that housing production is perfectly competitive. Let $\overline{u} = 3$. Let the household's problem be given by

$$\max_{c,h,x} c^{1/2}h^{1/2} \text{ subject to } w = c + ph + 2tx$$

Let $\widetilde{w} = w - 2tx$. Use the first-order condition of the household's problem with respect to h to find h^* in terms of p and $\widetilde{w}$.

The household's problem is:

$$\max_{c,h,x} c^{1/2}h^{1/2} \text{ subject to } \widetilde{w} = c + ph$$

Solving the constraint for c, we have $c = \widetilde{w} - ph$. If we plug this into the expression that we want to maximize, we no longer need to worry about the constraint, but instead can just solve:

$$\max_{h} (\widetilde{w} - ph)^{1/2}h^{1/2}$$

We do this, finding the value of h^* that maximizes the above expression, by setting the derivative of this expression with respect to h equal to zero.

$$\begin{aligned}
\frac{\partial[(\widetilde{w} - ph)^{1/2}h^{1/2}]}{\partial h} &= 0 \\
-(1/2)p(\widetilde{w} - ph^*)^{-1/2}h^{*1/2} + (1/2)(\widetilde{w} - ph^*)^{1/2}h^{-*1/2} &= 0 \\
(1/2)p(\widetilde{w} - ph^*)^{-1/2}h^{*1/2} &= (1/2)(\widetilde{w} - ph^*)^{1/2}h^{*-1/2} \\
ph^* &= \widetilde{w} - ph^* \\
2ph^* &= \widetilde{w} \\
h^* &= \frac{\widetilde{w}}{2p}
\end{aligned}$$

(b) Use the fact that utility is $\overline{u} = 3$ everywhere to solve for p^* in terms of $\widetilde{w}$.

$$
\begin{aligned}
\overline{u} &= 3 \\
\overline{u} &= (\widetilde{w} - p^* h^*)^{1/2} h^{*1/2} \\
&= \left(\widetilde{w} - p^* \frac{\widetilde{w}}{2p^*}\right)^{1/2} \left(\frac{\widetilde{w}}{2p^*}\right)^{1/2} \\
&= \left(\frac{\widetilde{w}}{2}\right)^{1/2} \left(\frac{\widetilde{w}}{2p^*}\right)^{1/2} \\
&= \sqrt{\frac{\widetilde{w}^2}{2^2 p^*}} \\
3 &= \frac{\widetilde{w}}{2\sqrt{p^*}} \\
\sqrt{p^*} &= \frac{\widetilde{w}}{2 * 3} \\
p^* &= \frac{\widetilde{w}^2}{36}
\end{aligned}
$$

(c) Substitute your expressions for p^* and $\widetilde{w}$ into your expression for h^* to write h^* in terms of w, t, and x.

$$
\begin{aligned}
h^* &= \frac{\widetilde{w}}{2p} \\
&= \frac{\widetilde{w}}{\widetilde{w}^2/18} \\
&= \frac{18}{\widetilde{w}} \\
&= \frac{18}{w - 2tx}
\end{aligned}
$$

(d) Let the developer's problem be given by

$$\max_S pS^{2/3} - iS - R$$

where S is the capital to land ratio, and p, i and R are the costs of housing, capital, and land, respectively. For the remainder of the problem, let $i = \frac{1}{33}$.
Comment: The technology for producing housing is constant returns to scale and can be written as $h_s(S) = S^{2/3}$. Here, h_s is housing supplied, and is (with constant returns to scale) units of housing per constant area. This is NOT the same as h in the household problem, which is housing units per person.
Use the first-order condition of this problem with respect to S to solve for ${h_s}^*$ in terms of p.

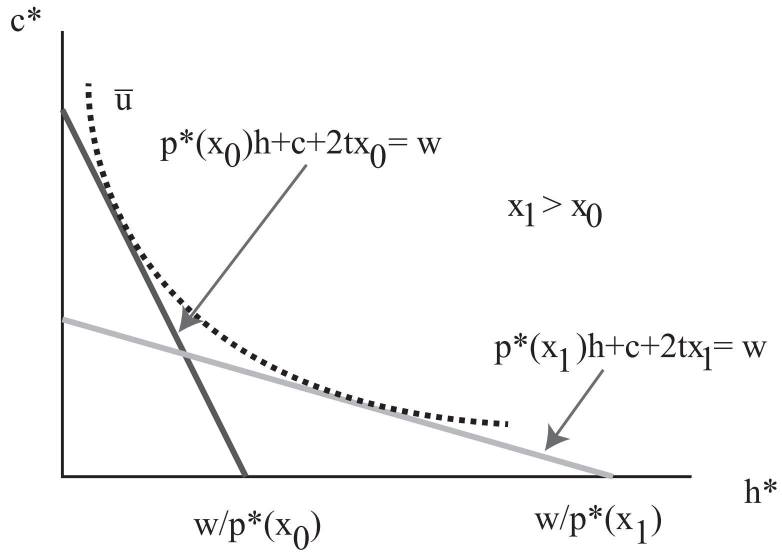
$$\frac{\partial(pS^{2/3} - iS - R)}{\partial S} = (2/3)pS^{-1/3} - i$$
$$(2/3)p^*S^{*-1/3} - i = 0$$
$$(2/3)p^*S^{*-1/3} = i$$
$$S^{*-1/3} = \frac{1.5i}{p^*}$$
$$S^* = \left(\frac{p^*}{1.5/33}\right)^3 = (22p^*)^3$$
$${h_s}^* = (S^*)^{(2/3)} = ((22p^*)^3)^{(2/3)} = (22p^*)^2$$

(e) The capital-land ratio S is our stylized description of building height. From part (d), $S^* = (22p^*)^3$. To figure how this changes with distance to the center, we differentiate with respect to x.

$$\frac{\partial(22p^*)^3}{\partial x} = 3(22p^*)^2 \times 22\frac{dp^*}{dx}$$

We know from part (b) that $\frac{dp^*}{dx} < 0$, so building height decreases with distance to the center.

3. Graphical analysis of the monocentric city model with housing



x and y axes are housing and composite consumption. The dashed curve is an indifference curve describing the choices of housing and consumption that satisfy the free mobility condition, $u(c,h) = \overline{u}$. The dark gray line is a budget line for a household nearer the CBD, and the light gray line is a budget line for a more remote household. If households are rational, their budgets must be tangent to their indifference curve, as drawn. Because the more remote household spends more on commuting, and so the y-intercept of their budget line is lower, the budget line for more remote household can only be tangent to their indifferent curve if it is less steep. A flatter budget line for the more remote household requires that housing prices fall with distance to the center.

4. In this problem, we will analyze property taxes in the monocentric city model.

(a) Assume we have an open, linear city with property tax rate τ_0. $R_0(x)$ is the land rent in this city. Set up the household's problem (you don't need to solve it).

The household's problem is:

$$\max_{x,c} u(c) \text{ such that } w = c + (1+\tau_0)R_0(x)\bar{l} + 2t|x|$$

(b) Assume the tax rate increases from τ_0 to τ_1, where $1+\tau_1 = (1.10)(1+\tau_0)$. Set up the household's problem with this new tax rate.

The household's problem is now:

$$\max_{x,c} u(c) \text{ such that } w = c + (1+\tau_1)R_1(x)\bar{l} + 2t|x|$$

(c) Using what you know about c^* in an open city equilibrium, solve for $R_1(x)$ in terms of $R_0(x)$. How did rental prices change when the property tax increased? How does the sum of rent and property taxes change?

In an open city equilibrium, $u(c^*) = \overline{u}$ everywhere, so $c^* = u^{-1}(\overline{u})$ in both tax regimes. That means $w - c^*$ is the same in both cases, so we can equate:

$$\begin{aligned}
(1+\tau_0)R_0(x)\bar{l} + 2t|x| &= (1+\tau_1)R_1(x)\bar{l} + 2t|x| \\
R_1(x) &= \frac{(1+\tau_0)R_0(x)\bar{l}}{(1+\tau_1)\bar{l}} \\
R_1(x) &= R_0(x)\frac{1+\tau_0}{1+\tau_1} \\
R_1(x) &= \frac{R_0(x)}{1.1}
\end{aligned}$$

Rental prices decreased by slightly less than 10% when the property tax rate increased.

The sum of rent and property taxes are:

$$\begin{aligned}
R_1(x) + \tau_1 R_1(x) &= R_1(x)(1+\tau_1) \\
R_0(x) + \tau_0 R_0(x) &= R_0(x)(1+\tau_0)
\end{aligned}$$

Combining what we have recently shown, we know that:

$$\begin{aligned}
R_1(x) &= R_0(x)\frac{1+\tau_0}{1+\tau_1} \\
R_1(x)(1+\tau_1) &= R_0(x)(1+\tau_0)
\end{aligned}$$

Therefore the sum of rent and property taxes stays the same after the tax rate increase.

(d) Suppose landlords are responsible for paying the property tax. What does this suggest about the relationship between what tenants pay and property taxes?

Because changes to the tax rate cannot change the consumption level, c^* that all households receive, the function of a property tax is to divide the land rent between the landlord and the tax collector. Changes to the tax rate cannot change the sum of what the tenant pays the landlord and what the tax collector gets. If the landlord is nominally responsible for paying the property tax, then the total rent the landlord collects (before paying the tax collector) does not vary with the tax rate.