EC1410 – Spring 2024

Midterm
8:30-9:30am, March 13, 2024
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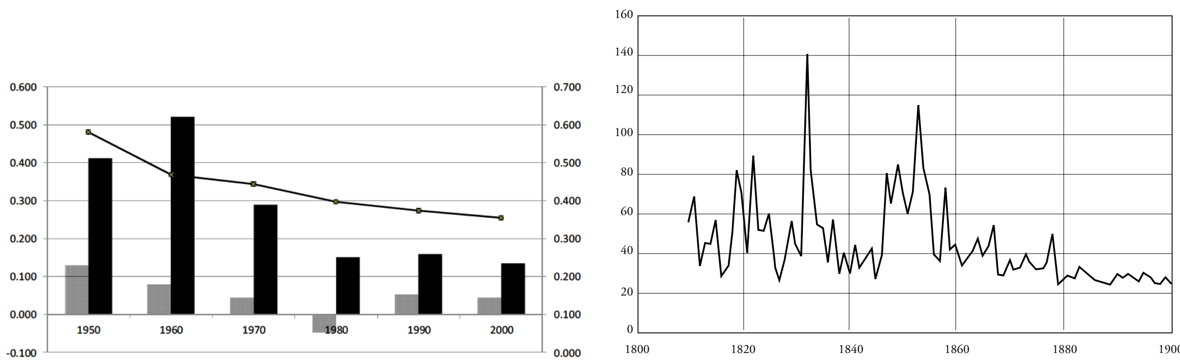
You will have 60 minutes to complete this exam. No notes or books are allowed, but you may use a calculator. Cell phones and any device with a wireless connection must be off. Anyone still working on their exam after time is called will be subject to an automatic 10 point penalty.

When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second goal, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Total points =100. Points assigned to each problem are indicated in parentheses.

This exam has TWO pages.

1. (a) (5) Panel (a) of the figure below is from Boustan, Bunten, and Heary's 2013 *NBER* working paper, 'Urbanization in the United States, 1800-2000'. What does the height of the heavy black bars represent?

   (b) (5) Panel (b) of the figure below is from Michael Haines' 2001 paper in the *Annales de Demographie Historique*, 'The Urban Mortality Transition in the United States, 1800-1940'. What are the units on the y axis? For what city?



(a) (b)

2. This problem asks you to work through an example of the monocentric city model. Assume we have a linear, open city. Let w=10, $\bar{l} = 1$, $\overline{R}$ = 0, $\overline{u}$ = 1, and $u(c) = c^{\frac{1}{2}}$.

   (a) (10) Find the edge of the city $\overline{x}$.

   (b) (10) Find the rent gradient, $R(x)$.

   (c) (10) Draw a graph illustrating w, c^*, and $R(x)$.

   (d) (10) Explain why aggregate rent is an interesting measure of welfare.

3. This problem will examine the change in the rent and purchase price gradients during the COVID pandemic, as estimated by Gupta et al. (2021).

(a) (5) Before the pandemic, the rental price gradient was described by:

$$\ln R_0(x) = 7.6 - 0.04 \ln(x+1)$$

where x is distance from the city center. During the pandemic, the rental gradient changed to:

$$\ln R_1(x) = 7.5 - 0.004 \ln(x+1)$$

What are the monthly rental prices at $x = 0$, before and during the pandemic? What is the percent change in rent at $x = 0$?

(b) (5) The asset price gradient before the pandemic was

$$\ln P_0(x) = 13.2 - 0.127 \ln(x+1)$$

During the pandemic, this gradient changed to:

$$\ln P_1(x) = 13.15 - 0.115 \ln(x+1)$$

What are the asset prices at $x = 0$, before and during the pandemic? What is the percent change in asset price at $x = 0$?

(c) (10) Throughout the pandemic, people have speculated that COVID would be "the death of cities". What do your calculations suggest about this sort of speculation?

4. This problem asks you to work through the monocentric city model with housing to verify that population density is decreasing with distance to the center.

Consider the monocentric city model with housing. Assume an open city, and a perfectly competitive housing market. Suppose the outside option is $\overline{u} = 1$.

The household's problem is,

$$\max_{c,h,x} c^{1/2} h^{1/2}$$
$$\text{s. t. } w = c + ph + 2tx$$

(a) (10) Let $\widetilde{w} = w - 2tx$. Find the demand for housing, $h(p,\widetilde{w})$, and consumption, $c(p,\widetilde{w})$.

(b) (5) Use your results in part (a) to write utility in terms of p and $\widetilde{w}$. That is, find $V(p,\widetilde{w})$, the indirect utility function.

(c) (5) Use $V(p,\widetilde{w})$, the spatial equilibrium condition, and the definition of $\widetilde{w}$, to solve for the housing price gradient in terms of w, t and x.

(d) (5) Let S be the ratio of housing capital to land, and $h_s(S) = S^{2/3}$ the amount of housing supplied per unit. That is, h_s is the supply of housing at a location. Let the price of capital be $i = 1/10$ and the rent on a unit of land be R. Then the developer's profit maximization problem is,

$$\max_{S} pS^{2/3} - iS - R$$

Solve for S^* in terms of p.

(e) (5) Use your results above to solve for the population density gradient in terms of w, t and x. Determine whether this gradient is increasing or decreasing in distance to the center.

1. (a) THE BLACK BARS GIVE THE SHARE OF URBAN GROWTH THAT IS SUBURBAN.

(b) THE y-AXIS IS THE CRUDE DEATH RATE (DEATHS PER 1000) FOR NEW ORLEANS.

2. WE HAVE

$$w = 10, \quad \bar{l} = 1, \quad P_c = 1, \quad \bar{R} = 0, \quad \bar{u} = 1$$

AND $u(c) = c^{1/2}$

(a) FIND $\bar{x}$. IN EQUILIBRIUM, WE MUST HAVE

$$u(c) = \bar{u} \quad \text{ALL } x$$

$$\Rightarrow \quad c^{1/2} = 1 \quad \Rightarrow \quad c^* = 1.$$

THE HOUSEHOLD BUDGET CONSTRAINT IS

$$w = c^* + R(x)\bar{l} + 2tx$$

AT THE EDGE OF THE CITY, $R(\bar{x}) = \bar{R}$. USING THIS, WITH $\bar{R} = 0$, $c^* = 1$, $w = 10$, WE HAVE

$$10 = 1 + 2t\bar{x}$$

$$\Rightarrow \bar{x} = \frac{9}{2t}$$

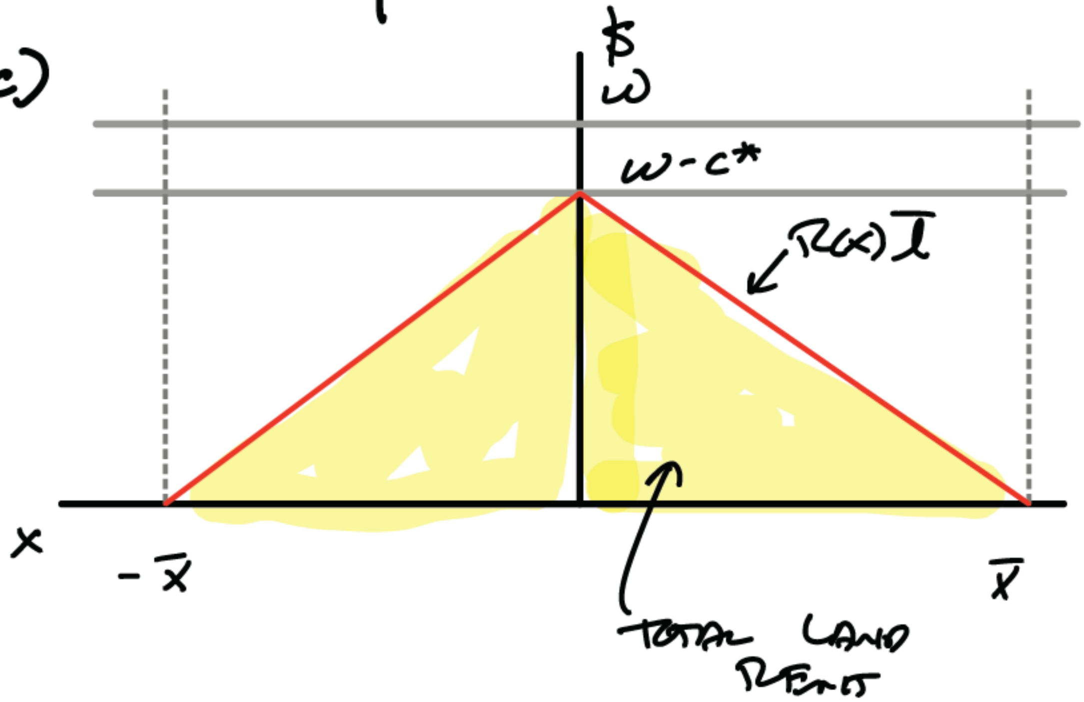
(b) FIND $R(x)$. FROM ABOVE, THE HOUSEHOLD BUDGET IS

$$w = c^* + R(x)\bar{l} + 2tx.$$

USING $w=10$, $c^*=1$, $\bar{l}=1$, WE HAVE $10 = 1 + R(x) + 2tx$ $(|x| < \bar{x})$

$$\Rightarrow \quad R(x) = \begin{cases} 9 - 2tx & |x| < 9/2t \\ 0 & |x| > 9/2t \end{cases}$$

(c)



(d) TOTAL LAND RENT IS HOW MUCH CITY RESIDENTS CAN GIVE UP, COLLECTIVELY, AND STILL BE INDIFFERENT BETWEEN THE CITY AND THEIR OUTSIDE OPTION. THUS, IT IS A MEASURE OF THE SURPLUS CREATED BY THE OPPORTUNITY TO COMMUTE TO $x=0$ AND EARN WAGE w.

(3) (a) WHEN $x=0$, $\ln(x+1) = \ln(1) = 0$,

SO $\ln(R_0) = 7.6 - 0.04 \ln(x+1)$

$= 7.6$

$\Rightarrow \quad R_0 = e^{7.6} \approx 1998\,\$$

SIMILARLY

$R_1 = e^{7.5} \approx 1808\,\$$

$$\frac{R_1}{R_0} = \frac{1808}{1998} \approx 0.9$$

SO RENTAL PRICES DECLINED BY 10% AT THE CITY CENTER DURING THE PANDEMIC

(b) THE LOGIC HERE IS JUST LIKE (a).

$\ln(P_0) = 13.2$

$\Rightarrow \quad P_0 = e^{13.2} = 540,364$

$\ln(P_1) = 13.15$

$\Rightarrow \quad P_1 = e^{13.15} = 514,011$

SO $$\frac{P_1}{P_0} = \frac{514,001}{540,346} = 0.95$$

(c) WE KNOW THAT ASSET PRICES AND RENTAL PRICES ARE MECHANICALLY RELATED. IF THE RENTAL PRICE DECLINE WERE PERMANENT, THEN WE WOULD NEED

$$1808 = r P_1^*$$

AND $1998 = r P_0$

THIS IS A DIFFERENT WAY TO SOLVE THIS THAN THE P.S. SOLUTION. THE OTHER WAY IS O.K. TOO

BUT THIS REQUIRED THAT ASSET PRICES FALL IN THE EXACT PROPORTION AS RENTAL PRICES. BECAUSE ASSET PRICES FELL BY LESS THAN RENTAL PRICES, IT MUST BE THAT PEOPLE EXPECT RENTAL PRICES WILL RISE.

(3) (a) WITH $\tilde{\omega} = \omega - \tau_x$, THE HOUSEHOLD PROBLEM IS

$$\max c^{1/2} h^{1/2}$$
$$\text{s.t. } \tilde{\omega} = c + ph \qquad *$$

$$\Rightarrow \max (\tilde{\omega} - ph)^{1/2} h^{1/2}$$

TO SOLVE, SET F.O.C. = 0

$$\Rightarrow \frac{1}{2}(\tilde{\omega}-ph)^{-1/2}(-p)h^{1/2} + \frac{1}{2}(\tilde{\omega}-ph)^{1/2}h^{-1/2} = 0$$

$$\Rightarrow (\tilde{\omega}-ph)^{-1/2}(-p)h^{1/2} + (\tilde{\omega}-ph)^{1/2}h^{-1/2} = 0$$

$$\Rightarrow -ph + (\tilde{\omega}-ph) = 0$$

$$\Rightarrow \tilde{\omega} = 2ph \Rightarrow h(p,\tilde{\omega}) = \frac{\tilde{\omega}}{2p} \qquad (**)$$

USING (*) $\Rightarrow$

$$\tilde{\omega} = c + p\left(\frac{\tilde{\omega}}{2p}\right)$$

$$\Rightarrow c(p,\tilde{\omega}) = \frac{\tilde{\omega}}{2} \qquad (***)$$

(b) USING (**) AND (***) IN THE UTILITY FUNCTION, WE HAVE

$$V(p,\tilde{\omega}) = h(p,\tilde{\omega})^{1/2}\, c(p,\tilde{\omega})^{1/2}$$

$$= \left(\frac{\tilde{\omega}}{2p}\right)^{1/2}\left(\frac{\tilde{\omega}}{2}\right)^{1/2}$$

$$= \frac{\tilde{\omega}}{2p^{1/2}}$$

(c) SPATIAL EQUILIBRIUM REQUIRES THAT

$$V(p,\tilde{\omega}) = 1$$ EVERYWHERE.

USING RESULTS FROM (b) THIS MEANS THAT

$$\frac{\tilde{\omega}}{2p^{1/2}} = 1$$

$$\Rightarrow \quad p^{1/2} = \frac{\tilde{\omega}}{2}$$

$$\Rightarrow \quad p = \frac{\tilde{\omega}^2}{4}$$

$$\Rightarrow \quad p(x) = \frac{(\omega - 2tx)^2}{4} \quad (\#)$$

(d) WE CAN USE (#) AND (**) TO WRITE HOUSING DEMAND AS A FUNCTION OF x.

$$h(p,\tilde{\omega}) = \frac{\tilde{\omega}}{2p} = \frac{(\omega - 2tx)}{2 \cdot \left(\frac{(\omega - 2tx)^2}{4}\right)}$$

$$= \frac{2}{\omega - 2tx} = h(x)$$

(e) THE DEVELOPER'S PROBLEM IS

$$\max_{S} \; pS^{2/3} - iS - R$$

$$\text{F.O.C.} = 0 \Rightarrow \quad \frac{2}{3} p S^{-1/3} = i$$

$$\Rightarrow \quad S^{-1/3} = \frac{3i}{2p}$$

$$\Rightarrow \quad S = \left(\frac{2p}{3i}\right)^3$$

$$\Rightarrow \quad S = \left(\frac{2\Phi p}{3}\right)^3 \quad (\#\#)$$

(f) POPULATION DENSITY IS HOUSING PER UNIT AREA h_S DIVIDED BY HOUSING PER PERSON h (**)

SO, USING (**) AND (##)

$$D = \frac{\left[\left(\frac{2\Phi p}{3}\right)^3\right]^{2/3}}{\frac{\tilde{\omega}}{2p}} = \frac{\left(\frac{2\Phi p}{3}\right)^2}{\frac{\tilde{\omega}}{2p}}$$

$$\Rightarrow D = \frac{2p \cdot (20p)^2}{3^2 \; \tilde{\hat{\omega}}}$$

$$= \frac{800 p^3}{9 \; \tilde{\hat{\omega}}} = \frac{800}{9} \cdot \frac{1}{\omega - 2tx} \cdot \left[\frac{(\omega - 2tx)^2}{4} \right]^3$$

$$= \frac{800}{9} \cdot \frac{1}{64} \cdot \frac{(\omega - 2tx)^6}{\omega - 2tx}$$

$$\Rightarrow D(x) = \frac{25}{18} \cdot (\omega - 2tx)^5$$

so $$\frac{d \, D(x)}{dx} = 5 \cdot \frac{25}{18} (\omega - 2tx)^4 (-2t)$$

$$< 0$$