# EC1410 – Spring 2023

Midterm

8:30-9:30am, March 15, 2023

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*You will have 60 minutes to complete this exam. No notes or books are allowed, but you may use a calculator. Cell phones and any device with a wireless connection must be off. Anyone still working on their exam after time is called will be subject to an automatic 10 point penalty.*

*When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Total points =100. Points assigned to each problem are indicated in parentheses.*

***This exam has TWO pages.***

1. Consider the monocentric city model without housing. Let $w$ be the wage, and let $\bar{\ell} = 1$, $p_c$ = 1, $\overline{R}$ = 0, unit (one way) commmute costs $t$ be 1/2, and $u(c) = c^{1/3}$.

   (a) Assume that the city is open to migration and that $\bar{u}$ = 1. Find,

      i. (5) $\bar{x}$, the distance from the center to the edge of the city.

      ii. (5) R(x) the equilibrium land rent gradient.

      iii. (5) $N$, the equilibrium population of the city.

      iv. (5) Aggregate land rent as a function of the wage.

   (b) (10) Assume the city is closed to migration and that $N = 1$. Write aggregate land rent as a function of the wage.

   (c) (5) Starting in the early 1990s, China embarked on a series of economic reforms. During the following decades, urban wages increased dramatically and 'hukou' restrictions limiting migration to the cities were relaxed. Also during this time, (for several reasons) municipal authorities derived some personal benefit from increases in urban land prices. In light of your answers to the two questions above, explain briefly why relaxing hukou restrictions would have been in the interest of municipal authorities during this time.

2. This problem asks you to consider the effect of the pandemic on an open monocentric city. Assume an open monocentric city with $w = 10$, $\bar{\ell} = 1$, $p_c$ = 1, $\overline{R}$ = 0, $u(A(x)c) = \ln(A(x)c - 1)$ and commute costs $t$.

   Notice that amenities, $A(x)$, are allowed to vary with distance. Suppose that before the pandemic, $A_0(x) = 1$, and that after the pandemic $A_1(x) = 0.9 + \frac{x}{10}$. In words, before Covid, amenities are the same everywhere in the city, and after, they are relatively worse in the center and relatively better farther away.

   Suppose that one way transportation costs also fall, from $t$ to $t/2$ after Covid.

   (a) (5) Solve for the pre-covid rent gradient.

(b) (10) In this problem, equilibrium consumption post-covid will vary with distance to the center. Find the equilibrium consumption gradient $c(x)$

(c) (10) Solve for the post-covid rent gradient.

(d) (10) Draw a graph illustrating; wages, consumption, and rent gradients, pre- and post-covid.

(e) (5) Is this model consistent with changes in the rent gradient described in, *Flattening the Curve: Pandemic-Induced Revaluation of Urban Real Estate*, 2021, by Arpit Gupta Vrinda Mittal Jonas Peeters, and Stijn Van Nieuwerburgh?

3. (15) Consider the problem of a monocentric city with housing. In such a city, a household solves

$$\max_{c,h} u(c,h)$$
$$\text{s. t. } w = c + ph + 2tx$$

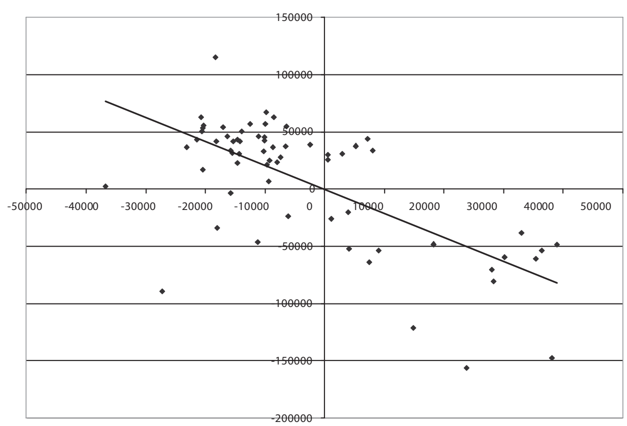
at each $x$. Suppose that indifference curves for $u$ are negatively sloped and convex (as we usually draw indifference curves), and that households at each $x$ obtain the same level of utility, $\overline{u}$. Provide a graphical proof that the price of housing decreases as the distance to the center increases.

4. (10) The following three questions ask for a short description of results presented in the course readings.

(a) In *The Economics of Urban Density*(2020), Gilles Duranton and Diego Puga list five benefits of higher urban densities. Name *exactly* two.

(b) In *The Urban Mortality Transition in the United States, 1800-1940*(2001) Michael Haines describes mortality in several US cities during the 19th century. What statistic does he use to measure mortality and how is it defined?

(c) The figure below is the first figure from *Was Postwar Suburbanization "White Flight"? Evidence from the Black Migration*(2010) Leah Platt Boustan. What do the $x$ and $y$ axes describe?



1. MONOCENTRIC CITY WITH $\bar{l}=1$, $R_c=1$, $\bar{R}=0$, $t=\frac{1}{2}$ AND $u(c)=c^{1/3}$.

(a) IF $\bar{u}=1$ THEN $u(c^*)=\bar{u}$

$$\Rightarrow \quad c^{*\,1/3}=1$$

$$\Rightarrow \quad c^*=1$$

SO MAX $u(c)$

S.T. $w=c^*+2tx+R(x)\bar{l}$ ①

(i) 5 WHEN $x=\bar{x}$, $R(x)=\bar{R}=0$

SO $w=c^*+2t\bar{x} \Rightarrow w=1+\bar{x}$

$$\Rightarrow \bar{x}=w-1$$

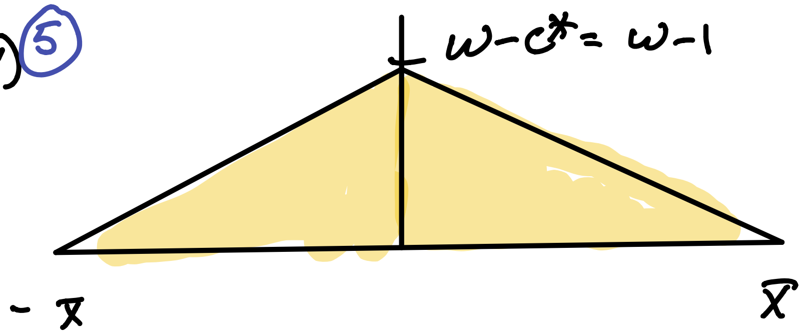
(ii) 5 FROM ① $w=c^*+2tx+R(x)\bar{l}$

$$\Rightarrow w=1+x+R(x)$$

$$\Rightarrow R(x)=\begin{cases} w-1-x & |x|<\bar{x} \\ 0 & |x|>\bar{x} \end{cases}$$

(iii) 5 WITH $\bar{l}=1$, $N=2\bar{x}=2(w-1)$

(iv) 5




AGGREGATE LAND RENT IS

$$2\left[\tfrac{1}{2}\left[(w-1)\bar{x}\right]\right]=(w-1)(w-1)$$

$$=(w-1)^2$$

(1 b) (10) IF $N=1$, THEN WITH $\bar{l}=1$, WE HAVE

$$2\bar{x}=\frac{N}{\bar{l}}=\frac{1}{1}=1 \Rightarrow \bar{x}=1/2$$ (2)

WE KNOW THAT AT $\bar{x}$

$$w = c^* + 2t\bar{x} + \bar{R}\,\bar{l}$$ (1)

WITH $t=1/2$, $\bar{R}=0$ AND $\bar{x}=1/2$ WE HAVE

$$w = c^* + 2\cdot 1/2 \cdot 1/2 + 0$$

$$\Rightarrow c^* = w - 1/2$$ (2)

THIS MEANS THAT

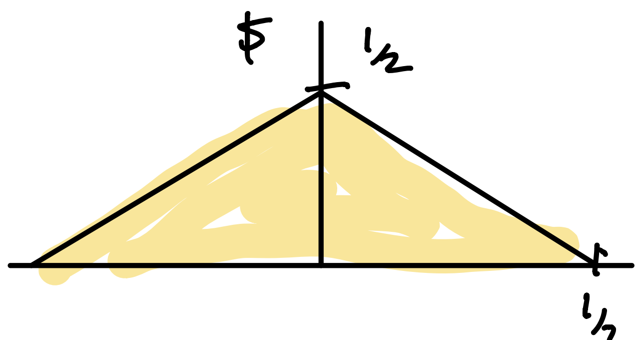
$$w = c^* + 2tx + R(x)\bar{l}$$

$$\Rightarrow w = (w-1/2) + x + R(x)$$

$$\Rightarrow R(x) = w - (w-1/2) - x$$ (2)

$$= \frac{1}{2} - x$$

SO WE HAVE $R(x)=\begin{cases} 1/2 - x & |x|<1/2 \\ & |x|>1/2 \end{cases}$




(3)

SO, AGGREGATE LAND RENT IS $2\times\frac{1}{2}\left(\frac{1}{2}\times\frac{1}{2}\right)=1/4$

(1c) IN AN OPEN CITY, AGGREGATE LAND RENT IS INCREASING IN THE WAGE. IN A CLOSED CITY IT IS NOT. IF A MUNICIPAL AUTHORITY BENEFITS FROM HIGHER LAND RENT HE WILL HAVE AN INCENTIVE TO MAKE A CITY MORE OPEN. THAT IS, TO RELAX HUKOU RESTRICTIONS.

5 2 3

2(a) WITH $\bar{u}=0$, WE HAVE

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$$0 = \ln(c-1) \Rightarrow c^* = e^0 = 1$$

FROM THE HOUSEHOLD BUDGET

$$w = c^* + 2tx + R(x)\bar{l}$$

SO

$$10 = 1 + 2tx + R(x)$$

$$\Rightarrow \quad 9 - 2tx = R^o(x) \quad , \quad |x| < \bar{x}$$

IS THE PRE-COND RENT GRADIENT

(b) WITH $\bar{u}=0$ WE HAVE

10

$$0 = \ln(A(x)c - 1)$$

$$\Rightarrow \quad A(x)c - 1 = 1$$

$$\Rightarrow \quad A(x)c = 2$$

$$\Rightarrow \quad c(x) = 2/A(x) = \frac{2}{.9 + x/10}$$

$$= \frac{20}{9+x}$$

SO $c(0) = \frac{20}{9}$ AND $c(x) \to 0$ AS $x \to \infty$

(c) FOR ALL $x$, WE MUST HAVE

10

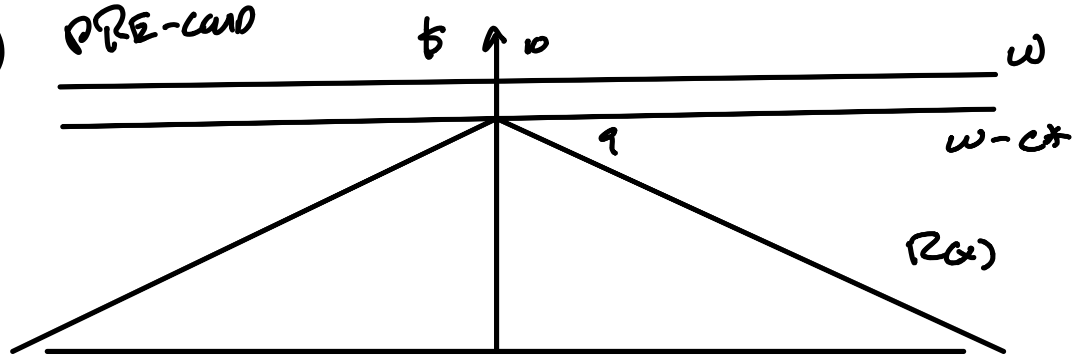
$$W = C^*(x) + 2\cdot\frac{t}{2}x + R(x)$$

$$10 = \frac{20}{9+x} + tx + R(x)$$

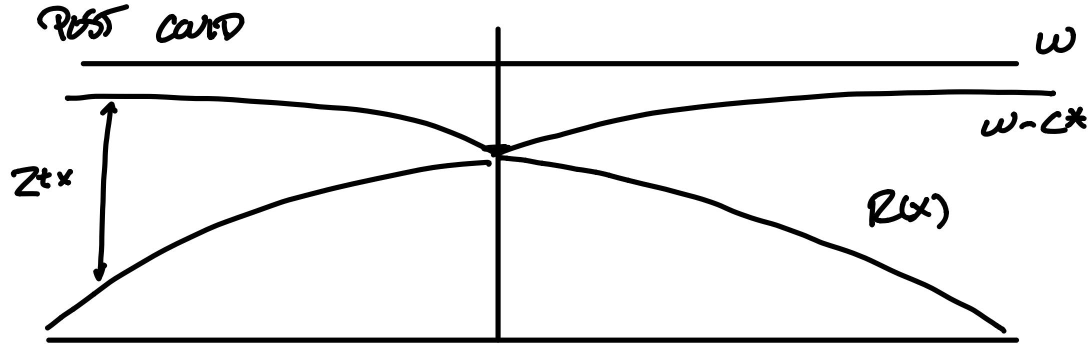
$$\Rightarrow R^1(x) = 10 - \frac{20}{9+x} - tx$$

(d) PRE-COND

10




POST COND




(e) NOTE THAT $R^1(0) = 10 - \frac{20}{9} < 9 = R^0(0)$

5

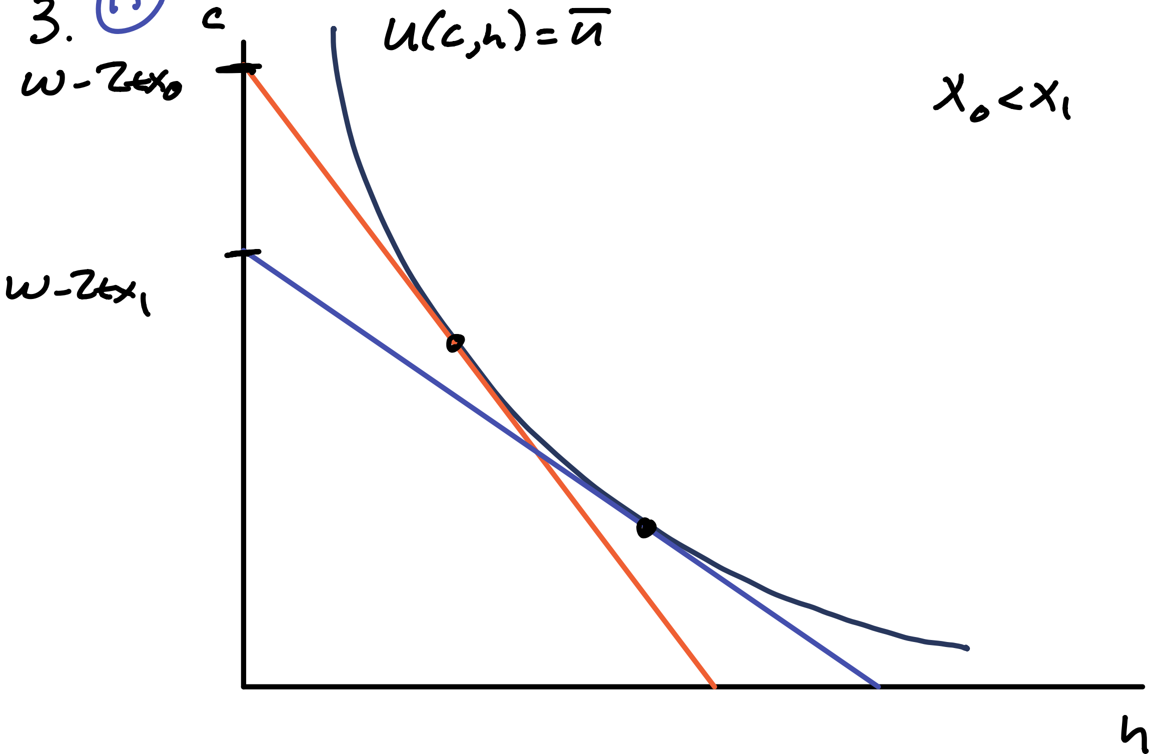
AND $R^1(x) = 10 - \frac{20}{9+x} - tx > 9 - 2tx = R^0(x)$

FOR $x$ LARGE ENOUGH

SO $R^1(x)$ STARTS BELOW $R^0(x)$ AT $x=0$ AND ENDS UP ABOVE IT.

THIS IS JUST THE PATTERN DISCUSSED IN THE PAPER. WITH COVID, CENTRAL RENTS DECLINED AND PERIPHERAL RENTS INCREASED.

3. 15




IF $h = 0$, THEN $c = w - 2tx$
THEREFORE, AS $x\uparrow$, THE y INTERCEPT OF THE BUDGET LINE MUST DECREASE

WITH OPTIMIZATION, WE MUST HAVE BUDGET LINES TANGENT TO $\bar{u}$. THEREFORE, AS $x\uparrow$ WE MUST MOVE FROM RED TO BLUE BUDGET LINE.

BUT $h_1 > h_0$ DESPITE MORE SPENDING ON COMMUTING $\Rightarrow$ $P_1 < P_0$ ■

4. (a) PRODUCTIVITY, ACCESSIBILITY, INNOVATION, REDUCED POLLUTION, AMENITIES.

5

(b) CRUDE DEATH RATE

5

$\equiv$

DEATHS PER 1000 OF POPULATION

(c) X-AXIS ~ BLACK MIGRATION

y-AXIS ~ WHITE MIGRATION.

5

EXTRA CREDIT