EC1410 – Spring 2022 Midterm 8:30-9:30am, March 16, 2022 Matt Turner

You will have 60 minutes to complete this exam. No notes or books are allowed but you may use a calculator. Cell phones and any device with a wireless connection must be off. Anyone still working on their exam after time is called will be subject to an automatic 10 point penalty.

When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Total points =100/Share of total grade =20%. Points assigned to each problem are indicated in parentheses.

This exam has TWO pages.

- 1. Consider the monocentric city model without housing. Let w = 10, $\overline{l} = 1$, $p_c = 1$, $\overline{R} = 1$, and $u(c) = c^{1/2}$.
 - (a) (20) Assume that the city is open to migration and that $\overline{u} = 1$. Solve for
 - i. \overline{x} , the distance from the center to the edge of the city.
 - ii. R(x) the equilibrium land rent gradient.
 - iii. *N*, the equilibrium population of the city.
 - (b) (20) Draw graph with x-axis distance to the CBD, and y-axis expenditure/consumption. Illustrate and label
 - i. Reservation level of consumption, c^* .
 - ii. Residential and rural land rent.
 - iii. Commute costs.
 - iv. Aggregate land rent.
 - v. Wage.
 - (c) (10) Assume that the city is closed to migration and that population is fixed at N = 10.
 - i. \overline{x} , the distance from the center to the edge of the city.
 - ii. R(x) the equilibrium land rent gradient.
 - iii. *N*, the equilibrium population of the city.
 - (d) (5) If wages increase, what will happen to land rent and utility in the open city? What about in the closed city?
 - (e) (5) Why is aggregate land rent a good measure of welfare in an open city?

- 2. Suppose that pre and post pandemic annual rental prices for an average house at the CBD are $R_0 = 24000$ and $R_1 = 21,600$ (where the '1' subscript is 'post', and the '0' subscript is 'pre'). Suppose that the cooresponding sale prices for these houses are $P_0 = 800,000$ and 780,000,
 - (a) (10) Find the percentage change in asset (sales) prices implied by the change in rental prices assuming an interest rate of $\rho = 0.03$.
 - (b) (10) Compare the percentage change you have just calculated to the realized price change in asset prices. Does this comparison suggest that the pandemic induced change in rental prices will be permanent? Explain briefly.
- 3. This problem will use what we know about the ratio of consumption between rural and urban areas in the developing world to determine what the implied ratio of rural to urban amenities must be according to the monocentric city model.

Henderson and Turner (2020) report that the density elasticity of household income in the developing world is around 32%. Using this estimate, moving from a density of 1,000 people per square kilometer to 20,000 people per square kilometer (the 10th and 80th percentiles of the density distribution in Sub-Saharan Africa) increases household income by a factor of about 2.6.

Let A_U and A_R be rural and urban amenities. Farmers do not commute and assume agricultural land rent is zero, so that $c_R^* = w_R$.

Assume an open monocentric city with w_R , w_U , $\overline{l} = 1$, $p_c = 1$, $\overline{R} = 0$, $u(Ac) = \ln(Ac - 1)$ and commute costs *t*.

- (a) (5) Assume use the requirement that utility be the same for urban and rural residents to find an expression for A_R/A_U in terms of w_R , and c_U^* .
- (b) (5) Use the monocentric city model to find an expression for c_U^* in terms of W_R , t, and \overline{x} .
- (c) (5) Suppose that households spend 1/2 of their income commuting (this is much greater than what we observe) so that $\frac{2tx}{w_U} = 0.5$. Use this assumption, together with your results above, to evaluate the fraction $\frac{A_R}{A_U}$.
- (d) (5) Does this ratio of rural to urban amenities make sense for the modern developing world? Explain briefly.

(1a) H.H. Saves Maxch 5.5. 10 = c + Z tx + R(x) IF U=1, THEN FREE MOBILITY REQUIRES C'2=1 => C*=1. THUS, THE H.H. BUDGET IS 10 = 1 + Z4x + RG) For Acc IxI<Z. AT THE EDGE OF THE CITY, X= R AND RG)=R=) => 10= 1 + ZEX + 1 => X= 4 IN SPATTAL EQUILIBRIUM, THE SAM OF REAT MID COMMUTTALE IS THE SAME AT ALL X. WITH 1x1 <7 THUS, ZAX + R(x) = ZAX + R =) Z+x+ R(x) = Z+ (4/2)+1 => Z+x+Rw) = 9 =) R(x) = 9-2+×

1

And so we have $T(w) = \begin{cases} 9 - 2tx & |x| < \frac{y}{k} \\ 1 & |x| > \frac{y}{k} \end{cases}$

TO FIND EQUICIBRIUM POPULATION, NOTE THAT THE CENICITY OF THE CITY 15



AGGREGATE LAND TRAT IS GREEN + PHARTE. AGGREGATE LAND TRAT CREATED TH TOX CMI IS PHARTE.

(1.) WITH POPULATION FIXED AT 10 AND $\overline{I} = I_{J}$ WE HAVE $10 = \frac{Z\overline{X}}{I} \implies \overline{X} = S$.

> SINCE THE SUM OF LAND RENT AND COMMUNING 13 CONSMANT, LEVE HAVE

$$Z + R = Z + R = |x| < \overline{x}$$

$$= 2 + x + R = |x| < \overline{x}$$

$$= 2 + x + R = |0 + 1|$$

$$= 10 + 1 - 2 + x = |x| < \overline{x}$$

And so we have

$$\begin{aligned}
& |c+1-2+x| & |x| < S \\
& |x| > S
\end{aligned}$$

$$\begin{aligned}
& |v| + |v| = |v| & |v| < |z| < |$$

=)
$$C^* = 9 - 10t$$

IT FOLLOWS THAT $U = U(C^*)$
= $(9 - 10t)^2$

(ld)

IF WAGES INCREASE IN THE OPENI CITY, LAND RENT WILL GO UP EXACTLY ENCLIGH TO LEEP CONSUMPTION CONSTRATE AT ALL THESIDENTIALX. SINCE THE PRICE OF CONSUMPTION IS CMIE, THIS MERANIS REFORT GOED UP EXACTLY AS MUCH AS THE LABLE. (Ie) AGGREGATE LAND REAT MEASURE THE TOTAL SURPLUS CARATRO TSY THE CITY. IT IS WHAT HOUSEHOLDS WILL PAY, IN THAT FOR THE OPPONTUNING TO STATLE IN THE CITY.

2. WE HAVE

$$R_0 = 24,000$$
, $R_1 = 2,600$
And $P_0 = 800,000$, $P_1 = 780,0000$.

(a) IF THE REMAR COST IS JUST THE INTEMENT ON THE ARSKET PRICE, THEN WE SHELLD HAVE

$$r\tilde{P}_{1} = R_{0} \implies \tilde{P}_{2} = \frac{24,000}{0.03} = 800,000$$

 $r\tilde{P}_{1} = R_{1} \implies \tilde{P}_{2} = \frac{21,600}{0.03} = 720,000$

3. IN SPATTAL EQUILIBRIUM, WE MUST

$$\mathcal{U}(A_n c_n^*) = \mathcal{U}(A_n c_n)$$

WE KNOW THAT CR = WR, SO WE HAVE

$$ln(A_nc^{*-1}) = ln(A_nu_R^{-1})$$

$$= \int A_{u} C_{u}^{*} - I = A_{p} W_{p} - I$$

$$= \int A_{u} C_{u}^{*} = A_{p} W_{p}$$

$$(3_{u}) = \int C_{u}^{*} = \frac{A_{p}}{A_{u}} W_{p}$$

$$(1)$$

$$IF WE CAH WRITE C* INI TENES OF
$$W_{u}, WE WRITE SE PONE.$$

$$USINIG THE H.H. TSUPGET CONSTRANT
For THE MUNICELITIE CHY MUNTEL,
$$W_{u}^{=} C_{u}^{*} + 24x + R(u) I$$

$$Ar THE EOGS OF THE CITY (SINUE R=0)$$

$$W_{u} = C_{u}^{*} + 24x = C (TY) (SINUE R=0)$$

$$W_{u} = C_{u}^{*} + 24x = 24x$$

$$W_{u} = C_{u}^{*} + 24x = 24x$$

$$W_{u} = C_{u}^{*} + 24x = 12$$

$$W_{u} = C_{u}^{*}$$$$$$

$$C_{n}^{*} = W_{n} - \frac{1}{2} w_{n}$$

$$\Longrightarrow \quad C_{n}^{*} = \frac{1}{2} w_{n}$$
RECALLING THAT $W_{n} = 2.6 W_{R}$,
 W_{E} HAVE $C_{n}^{*} = \frac{1}{2} (2.6 w_{R})$
 $\Longrightarrow \quad C_{n}^{*} = \frac{1}{2} (2.6 w_{R})$
 $\Longrightarrow \quad C_{n}^{*} = 1.3 w_{R}$.
SUBSTITUTING $\boxdot \quad \Longrightarrow \\ HAVE$
 $1.3 w_{R} = \frac{A_{R}}{A_{n}} \cdot w_{R}$
 $(3c) \implies \qquad A_{R} = 1.3$

(3) UNICIES THE DEVELOPED WINCO, DEVELOPING WINCO CATES DON'T SEEM TO TSE AS DANGEMONS. THUS, WE NEED SOME REASON OTHER THAN THE URTSAN MONTALITY REFENSION TO FEED REAR CUT OF CIDES. A RURA AMENDY PREMIUM CAN PO THIS.