EC1410 – Spring 2026

Final Exam

May 7, 2:00-3:15pm

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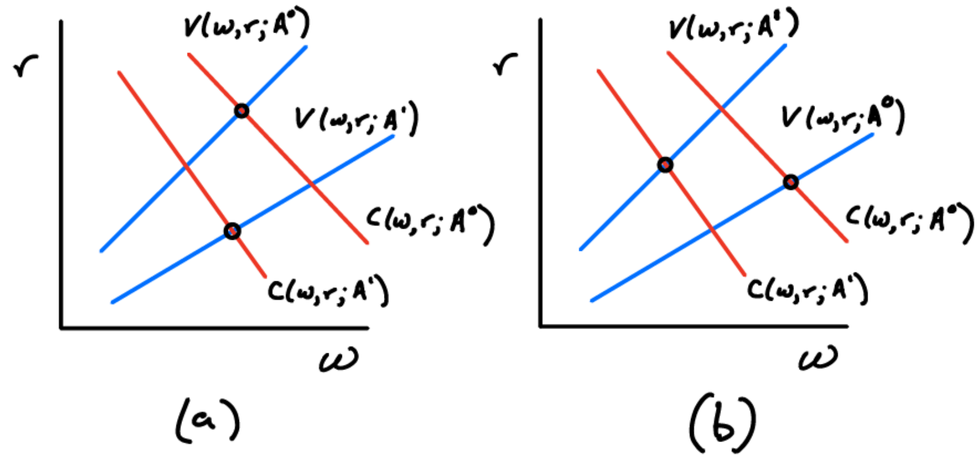
You will have 90 minutes to complete this exam. No notes or books are allowed but you may use a calculator. Cell phones and any device with a wireless connection must be off. Anyone still working on their exam after time is called will be subject to an automatic 10 point penalty.

When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Total points =100/Share of total grade =40%. Points assigned to each problem are indicated in parentheses.

We will mark these using gradescope. To facilitate this, please write your answers in the space provided. If you need more space use the blank pages at the back and ***make a note*** *telling us where to look.*

This exam has 15 pages.

1. (20) Consider the spatial equilibrium underlying the Roback Theorem. Let, $V(r,w;A)$ be the indirect utility function for city residents with arguments, w, wage, r, rent, and A, amenity. Let $C(w,r;A)$ be the unit cost function for the commercial activity in the city. Consider the following two figures showing indifference curves an iso-cost curves for two levels of an amenity, where $A_1 > A_0$.



(a) For the economy in panel (a), does the amenity increase or decrease productivity? Does it increase or decrease utility? Explain briefly.

(b) For the economy in panel (b), does the amenity increase or decrease productivity? Does it increase or decrease utility? Explain briefly.

2. This problem asks you to use the Roback Theorem to calculate the price of amenity A. Let w be the wage in a city (or location), r be land rent, ℓ_c be residential land consumption by a representative household in equilibrium and p_A the price of the amenity. Recall that the Roback Theorem states that

$$p_A = \ell_c \frac{dr}{dA} - \frac{dw}{dA}.$$

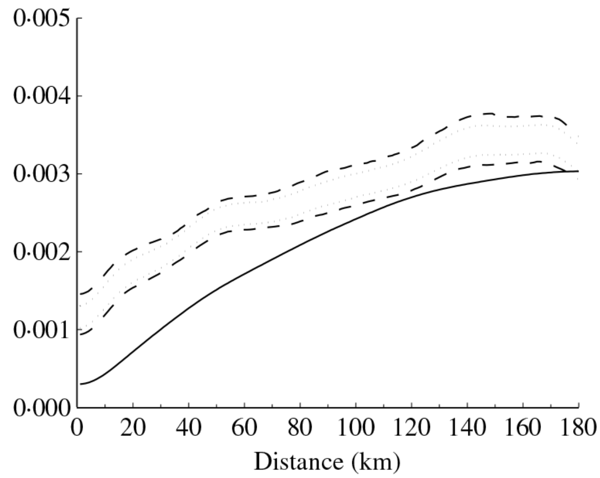
Assume you have data on rents, wages, and amenity A and have performed regressions of wages on amenities and of rents on amenities and produced the following estimates,

$$r = 1 + 2A$$
$$w = -6 - A$$

(a) (10) Use these regressions to estimate p_A.

(b) (10) Do you need more information to produce a numerical value for p_A? If so, what do you need to know?

3. (20) Consider the figure describing the distribution of pairwise distances between establishments producing other agricultural machinery from Duranton and Overman, 2005.



(c) Other Agricultural and Forestry Machinery (SIC2932)

Does this figure suggest that other agricultural machinery are more or less agglomerated than we expect if they chose their locations at random? Explain briefly.

4. In this problem, we consider the monocentric city model without housing, but two types of agents. The two types of agents are automobile drivers (a) and bus riders (b), with $w_a = 10$ and $w_b = 5$. Assume that transportation costs for the two types are

$$(w_b t^b + c^b)|x| \text{ for bus riders } t^b = 3, c^b = 1$$
$$f + (w_a t^a + c^a)|x| \text{ for drivers, where } t^a = 1, c^a = 2$$

In addition, assume $u(c) = \ln(c-1)$, $\overline{u} = 0$, $\overline{R} = 0$, $\overline{\ell} = 1$ and $p_c = 1$.

   (a) (5) Set up the household problem for each type of agent.

(b) (5) Assume $u(c^*) = \overline{u}$ for both types. Find c^*.

(c) (5) Find the bid rent functions $\Psi_a(x)$ and $\Psi_b(x)$, for both types.

(d) (5) Suppose $f = 5$ and plot $\Psi_a(x)$ and $\Psi_b(x)$ on one graph. Indicate the areas in which each type has the higher willingness to pay. Describe the resulting equilibrium briefly.

5. Consider a discrete linear city with three neighborhoods $i \in \{1, 2, 3\}$. Let x_i denote a neighborhood's distance from the CBD, with $x_1 = 1$, $x_2 = 2$, $x_3 = 3$. The cost to commute one unit distance is τ. The city is populated by households indexed by ν. Each household chooses a neighborhood i, pays land rent R_i, and commutes to the center, at location 0, to earn wage w. A household's utility is $V_i(\nu) = \varepsilon_i(\nu)c_i$ where c_i is consumption in location i, and $\varepsilon_i(\nu)$ is the household and location specific valuation. All $\varepsilon_i(\nu)$ are drawn from a Frechet distribution, $F(z) = e^{-T\varepsilon^{-\theta}}$.

   (a) (5) Let consumption be $c_i = w - R_i - i\tau$. Set up the household's problem.

(b) (5) The following theorem describes household behavior in discrete choice problems.

Theorem: *Suppose that households choose among N discrete locations. For each location $i = 1,...N$, household ν receives payoff $V_i(\nu) = \varepsilon_i u_i$, and ε_i is drawn from a Frechet distribution, $F(\varepsilon) = e^{-T\varepsilon^{-\theta}}$.*

Then the share of households such that

$$V_i(\nu) = max\{V_1(\nu),V_2(\nu),...,V_N(\nu)\}$$

is

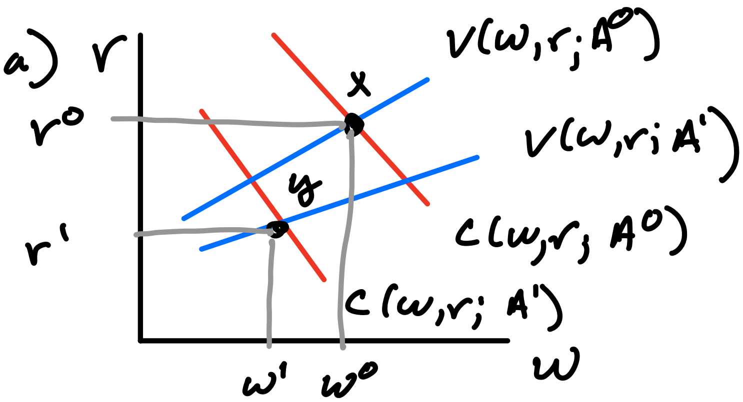
$$s_i = \frac{u_i^\theta}{\sum_{k=1}^N u_k^\theta}.$$

Use the this theorem to solve for the share of households s_i in each location.

(c) (5) Let the share of households in each location $s_1 = s_2 = s_3 = \frac{1}{3}$, wage $w = 6$ and the price of agricultural land $\bar{R} = 1$. Assume that the land rent at $x = 3$ is equal to $\bar{R} = 0$. Solve for R_1, R_2 and R_3 in terms of τ.

(d) (5) Plot land rent and commuting costs as a function of i.

(1a)



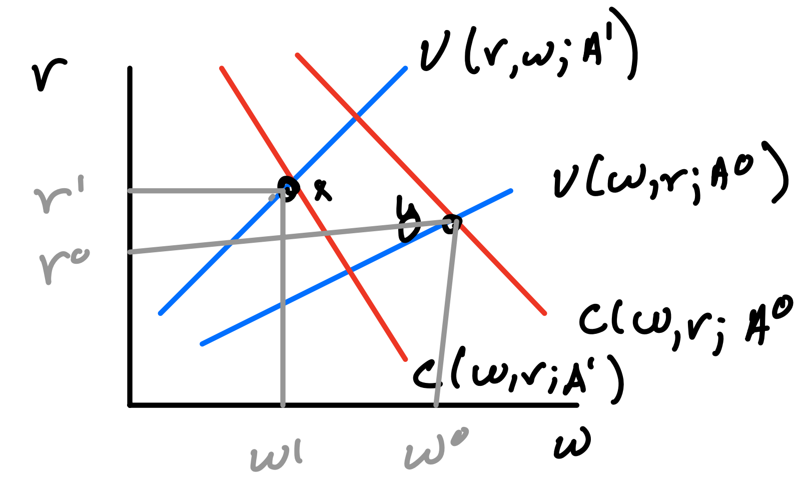
- WITH $A = A^1$, EQUILIBRIUM IS AT y
  $A = A^0$ x

(a) FIX WAGE AT w^0. HOUSEHOLDS ARE WILLING TO PAY r^0 FOR A^0, BUT ARE WILLING TO PAY <u>LESS</u> RENT FOR A^1 ⇒ H.H. PREFER A^0 TO A^1

- WITH $A^1 > A^0$ THIS MEANS A IS A BAD. HOUSEHOLDS WOULD RATHER HAVE LESS OF IT.

FIX WAGE AT w^0. FIRMS ARE WILLING TO PAY r^0 FOR A^0, BUT LESS FOR A^1.

- WITH $A^1 > A^0$, A MAKES FIRMS LESS PRODUCTIVE.



WITH A = A', EQUILIBRIUM IS AT X

A = A° y

(b) FIX W AT W'

H.H. WILL PAY r' FOR A', BUT LESS FOR A°. WITH A' > A° THIS MEANS A IS GOOD FOR H.H. THEY WILL PAY FOR MORE

FIRM WILL PAY r' FOR A', BUT <u>MORE</u> FOR A°. WITH A' > A° THIS MEANS MORE A MAKES FIRMS <u>LESS</u> PRODUCTIVE.

(2) (a) WE KNOW THAT

$$r = 1 + 2A \Rightarrow \frac{dr}{dA} = 2$$

$$w = -6 - A \Rightarrow \frac{dw}{dA} = -1$$

USING THE ROBACK FORMULA

$$P_A = l_c \cdot 2 - (-1)$$
$$= 2l_c + 1$$

(b) TO COMPUTE P_A, WE STILL NEED AN ESTIMATE OF PER HOUSEHOLD LAND/HOUSING CONSUMPTION.

(3) THE DASHED LINES GIVE US THE RANGE OF EXPECTED SHARES OF PAIRWISE DISTANCES BETWEEN ESTABLISHMENTS.

THAT THE SOLID LINE, ACTUAL SHARES, LIE BELOW THE EXPECTED RANGE TELLS US THERE ARE TOO FEW SHORT DISTANCE PAIRS RELATIVE TO RANDOMNESS

$\Longrightarrow$ THESE ESTABLISHMENTS PROBABLY TRY TO SPREADOUT.

(4) (a)

BUS RIDERS

$\text{MAX } u(c)$ $\quad 5 \cdot 3 + 1$

$\text{S.T. } \quad w^b = c + R(x)\bar{l} + [w^b t^b + c^b]|x|$

$\Longrightarrow \text{MAX } \ln(c-1)$

$\text{S.T. } \quad 5 = c + R(x) + 16|x|$

DRIVERS

$\text{MAX } u(c)$ $\quad 10 \cdot 1 + 2$

$\text{S.T. } \quad w^a = c + R(x)\bar{l} + [w^a t^a + c^a]|x| + f$

$\Longrightarrow \text{MAX } \ln(c-1)$

$\text{S.T. } \quad 10 = c + R(x) + 22|x| + f$

(b) WITH $\bar{u} = 0$ $\quad c^*$ SATISFIES $\ln(c^* - 1) = 0$

$\Longrightarrow c^* = 2.$

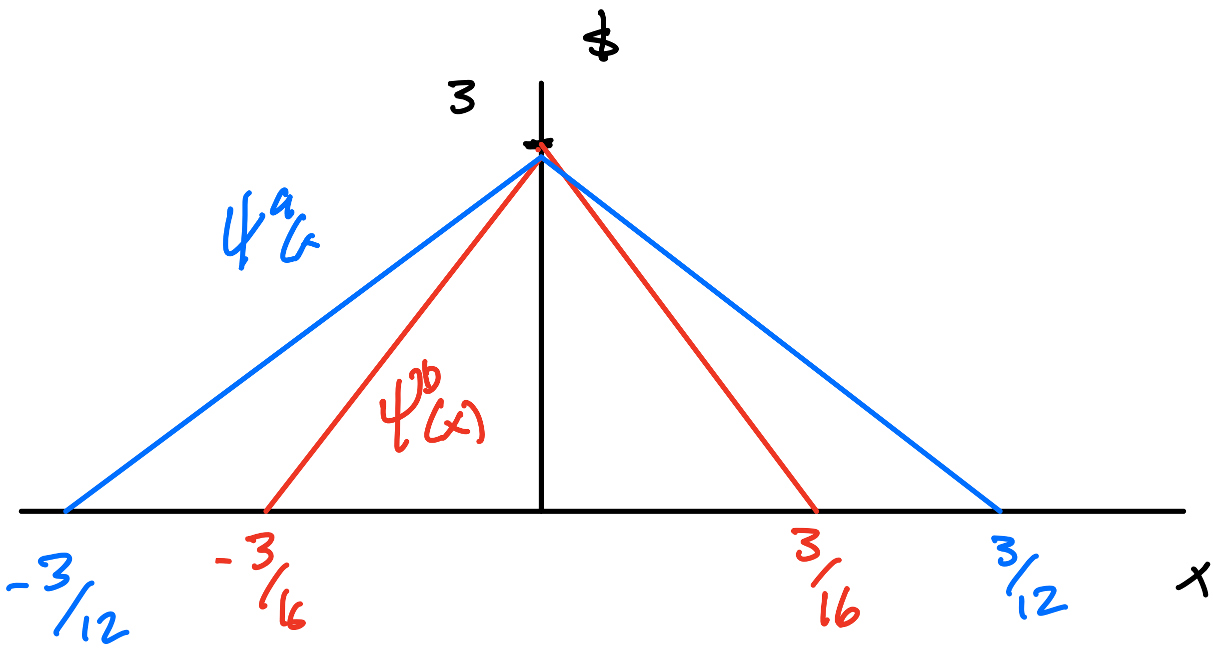
(c) $\psi^b(x) = 5 - 2 - 16|x|$

$= 3 - 16|x|$

$\psi^a(x) = 10 - 2 - 12|x| - f$

$= 8 - f - 12|x|$

(d) WITH $f = 5$, $\psi^a(x) = 3 - 12|x|$



SO DRIVERS OUTBID BUS RIDERS EVERYWHERE EXCEPT AT THE VERY CENTER, $x = 0$. THE CITY IS ALMOST ENTIRELY DRIVERS.

(5) (a) $\text{MAX} \quad \varepsilon_i(r)\, c_i$

S.T. $\quad c_i = w - R_i - i\tau$

(b) USING THE BUDGET WE HAVE

$$c_1 = w - R_1 - \tau$$

$$c_2 = w - R_2 - 2\tau$$

$$c_3 = w - R_3 - 3\tau$$

SO HOUSEHOLD TYPE r SOLVES

$$\text{MAX} \left\{ \varepsilon_1(r) c_1 ,\ \varepsilon_2(r) c_2 ,\ \varepsilon_3(r) c_3 \right\}$$

AND USING THE THEOREM, WE GET:

$$s_1 = \frac{(w - R_1 - \tau)^\theta}{(w - R_1 - \tau)^\theta + (w - R_2 - 2\tau)^\theta + (w - R_3 - 3\tau)^\theta}$$

$$s_2 = \frac{(w - R_2 - 2\tau)^\theta}{[\qquad\qquad]}$$

$$s_3 = \frac{(w - R_3 - 3\tau)^\theta}{[\qquad\qquad]}$$

(c) WITH $S_1 = S_2 = S_3 = 1/3$ WE MUST HAVE THE NUMERATORS OF THE S_i THE SAME $\Rightarrow$

$$w - R_1 - \tau = w - R_2 - 2\tau = w - R_3 - 3\tau$$

$$\Rightarrow \quad R_1 + \tau = R_2 + 2\tau \Rightarrow R_1 = R_2 + \tau$$

AND

$$\Rightarrow \quad R_2 + 2\tau = R_3 + 3\tau \Rightarrow R_2 = R_3 + \tau$$

BUT $R_3 = \bar{R} = 0$ SO WE HAVE

$$(R_1, R_2, R_3) = (2\tau, \tau, 0)$$

(d)