EC1410 – Spring 2025

Final Exam
May 5, 2:00-3:15pm
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You will have 75 minutes to complete this exam. No notes or books are allowed but you may use a calculator. Cell phones and any device with a wireless connection must be off. Anyone still working on their exam after time is called will be subject to an automatic 10 point penalty.

When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Total points =100/Share of total grade =40%. Points assigned to each problem are indicated in parentheses.

This exam has THREE pages.

1. (20) This problem asks you to use the Roback Theorem to calculate the importance of amenity A in real terms. Let w be the wage in a city (or location), r be land rent, ℓ_c be residential land consumption by a representative household in equilibrium and p_A the price of the amenity. Recall that the Roback Theorem states that

   $$p_A = \ell_c \frac{dr}{dA} - \frac{dw}{dA}.$$

   Assume you have data on rents, wages, and amenity A for a cross-section of cities. That is, your data is $\{r_i, w_i, A_i\}$ for a set of cities $i = 1, ..., J$. Assume that you also observe ℓ_c.

   Describe how to use these data to estimate p_A.

2. In this problem, we consider the monocentric city model without housing, but two types of agents. The two types of agents are automobile drivers (a) and bus riders (b), with $w_a = 10$ and $w_b = 5$. Assume that transportation costs for the two types are

   $$(w_b t^b + c^b)|x| \text{ for bus riders } t^b = 2, c^b = 1$$
   $$f + (w_a t^a + c^a)|x| \text{ for drivers, where } t^a = 1, c^a = 2$$

   In addition, assume $u(c) = \ln(c-1)$, $\overline{u} = 0$, $\overline{R} = 0$, $\overline{\ell} = 1$ and $p_c = 1$.

   (a) (5) Set up the household problem for each type of agent.

   (b) (5) Assume $u(c^*) = \overline{u}$ for both types. Find c^*.

   (c) (5) Find the bid rent functions $R_a(x)$ and $R_b(x)$, for both types.

   (d) (5) Suppose $f = 5$ and plot $R_a(x)$ and $R_b(x)$ on one graph. Indicate the areas in which each type has the higher willingness to pay. Describe the resulting equilibrium briefly.

3. In this problem, we will analyze property taxes in the monocentric city model without housing.

   (a) (5) Assume we have an open, linear city with property tax rate τ_0. $R_0(x)$ is the land rent in this city. Set up the household's problem (you don't need to solve it).

   (b) (5) Assume the tax rate increases from τ_0 to τ_1, where $1 + \tau_1 = (1.10)(1 + \tau_0)$. Set up the household's problem with this new tax rate.

   (c) (5) Using what you know about c^* in an open city equilibrium, solve for $R_1(x)$ in terms of $R_0(x)$. How does the sum of rent and property taxes change as the property tax changes?

   (d) (5) If an objective of the city government is to create affordable rental housing, does this example suggest that lowering the property tax is a good idea?

4. Consider a discrete linear city with three neighborhoods $i \in \{1, 2, 3\}$. Let x_i denote a neighborhood's distance from the CBD, with x_1 = 1, x_2 = 2, x_3 = 3. The cost to commute one unit distance is τ. The city is populated by households indexed by ν. Each household chooses a neighborhood i, pays land rent R_i, and commutes to the center, at location 0, to earn wage w. A household's utility is $V_i(\nu) = \varepsilon_i(\nu)c_i$ where c_i is consumption in location i, and $\varepsilon_i(\nu)$ is the household and location specific valuation. All $\varepsilon_i(\nu)$ are drawn from a Frechet distribution, $F(z) = e^{-T\varepsilon^{-\theta}}$.

   (a) (5) Let consumption be $c_i = w - R_i - i\tau$. Set up the household's problem.

   (b) (5) The following theorem describes household behavior in discrete choice problems.

   Theorem: *Suppose that households choose among N discrete locations. For each location $i = 1,...N$, household ν receives payoff $V_i(\nu) = \varepsilon_i u_i$, and ε_i is drawn from a Frechet distribution, $F(\varepsilon) = e^{-T\varepsilon^{-\theta}}$.*

   Then the share of households such that

   $$V_i(\nu) = max\{V_1(\nu), V_2(\nu), ..., V_N(\nu)\}$$

   is

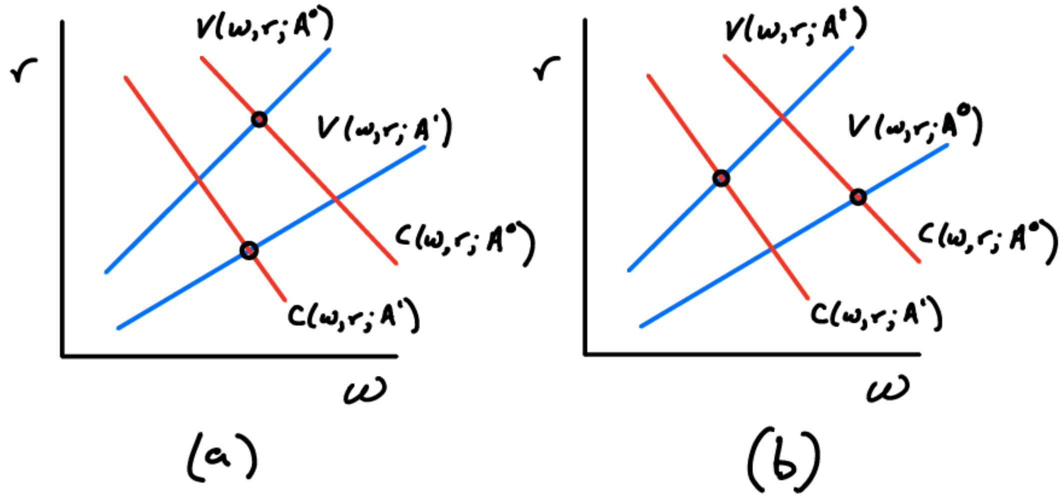
   $$s_i = \frac{u_i^\theta}{\sum_{k=1}^N u_k^\theta}.$$

   Use the this theorem to solve for the share of households s_i in each location.

   (c) (5) Let the share of households in each location $s_1 = s_2 = s_3 = \frac{1}{3}$, wage $w = 5$ and the price of agricultural land $\bar{R} = 1$. Assume that the land rent at $x = 3$ is equal to R. Solve for R_1, R_2 and R_3 in terms of τ.

   (d) (5) Plot land rent and commuting costs as a function of i.

5. Consider the following two figures showing indifference curves and iso-cost curves for two levels of an amenity, where $A_1 > A_0$.



(a) (10) For the economy in panel (a), does the amenity increase or decrease productivity? Does it increase or decrease utility and utility? Explain briefly.

(b) (10) For the economy in panel (b), does the amenity increase or decrease productivity? Does it increase or decrease utility and utility? Explain briefly.

1. GIVEN DATA $\{r_i, w_i, A_i\}$ FOR CITIES $i = 1, \dots, J$, WE CAN RUN THE FOLLOWING TWO REGRESSIONS

$$r_i = B_0 + B_1 A_i + \varepsilon_i$$

$$w_i = C_0 + C_1 A_i + \eta_i$$

THIS GIVES ESTIMATES OF B_1 AND C_1

DIFFERENTIATING THE REGRESSION EQUATIONS WITH RESPECT TO A_i, WE SEE THAT

$$\frac{dr_i}{dA_i} = B_1 \qquad \frac{dw_i}{dA_i} = C_1$$

USING THIS IN ROBACK'S FORMULA WE CAN ESTIMATE P_A AS

$$P_A = l_c B_1 - C_1$$

2. WE HAVE $w_a = 10$, $w_b = 5$, $t^b = 2$, $c^b = 1$, AND, $t^a = 1$, $c^a = 1$.

A BUS RIDERS PROBLEM IS

$$\text{MAX } c$$
$$\text{S.T. } w^b = c + (w^b t^b + c^b)|x| + R\, l_c$$

USING DATA GIVEN IN THE PROBLEM THIS BECOMES

$$\text{MAX } \ln(c-1)$$
$$\text{S.T. } 5 = c + (5 \cdot 2 + 1)|x| + R$$

OR

$$\text{MAX } \ln(c-1)$$
$$\text{S.T. } 5 = c + 11x + R$$

WITH $\bar{u} = 0$, WE HAVE $\ln(c^* - 1) = 0$

$$\Rightarrow c^* - 1 = e^0$$
$$\Rightarrow c^* = 2$$

$$\Rightarrow 5 = 2 + 11x + R$$
$$\Rightarrow R^b = 3 - 11x$$

A DRIVERS PROBLEM IS

$$\text{MAX } \ln(c-1)$$
$$\text{S.T. } w^a = c + f + (w^a t^a + c^a)|x| + R\, l_c$$

USING $c^* = 2$ AND OTHER DATA, WE HAVE

$$10 = 2 + f + (10 \cdot 1 + 2) 1 \times 1 + R$$

$$\Rightarrow \quad 8 = f + 12x + R$$

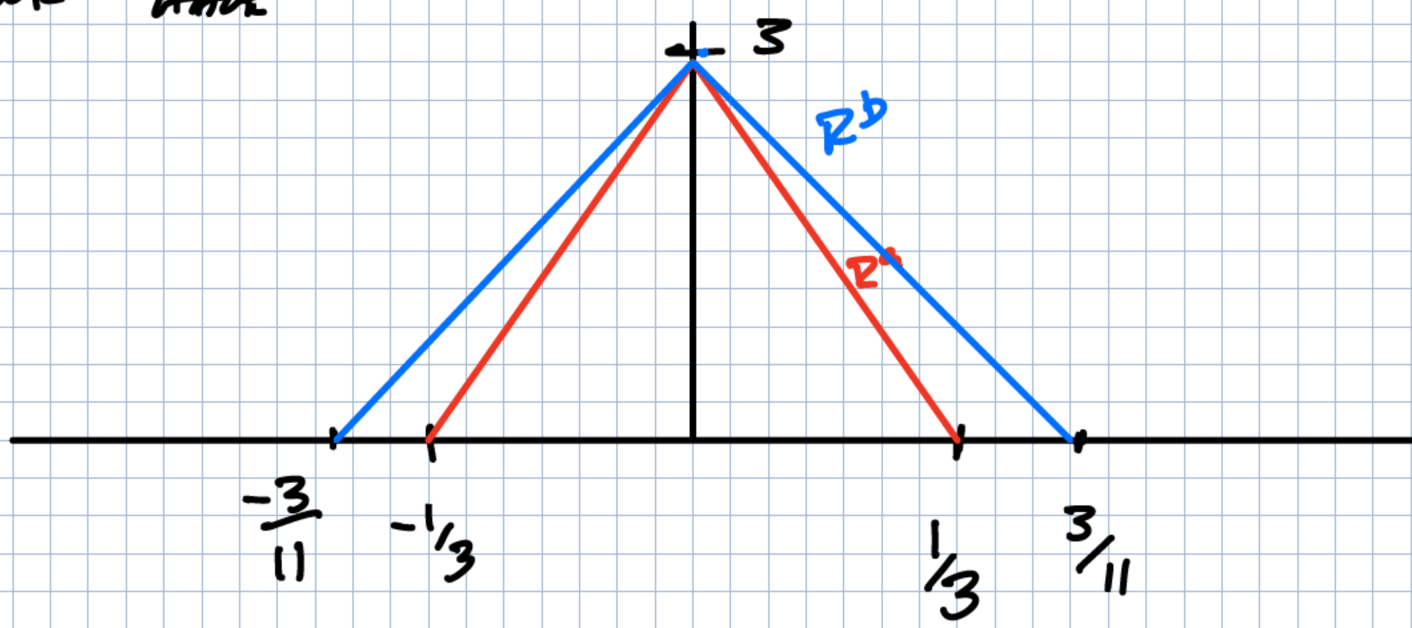
$$\Rightarrow \quad R^a = 8 - f - 12x$$

IF $f = 5$ THEN

$$R^a = 3 - 12x$$

SO WE HAVE $R^a = 3 - 12x$ AND $R^b = 3 - 11x$

IF WE PLOT THESE TWO RENT CURVES WE HAVE



SO BUS RIDERS OUTBID DRIVERS EVERYWHERE EXCEPT RIGHT AT ZERO (WHERE THERE IS NO COMMUTING). THIS IS A CITY WITH ONLY BUS RIDERS.

3. (a) $\max\ u(c)$

$$\text{s.t.}\quad w = c + zt|x| + (1+\tau_0)R_0 l_c \qquad (1)$$

(b) $\max\ u(c)$

$$\text{s.t.}\quad w = c + zt|x| + (1+\tau_1)R_1 l_c \qquad (2)$$

$$= \quad \max\ u(c)$$

$$\text{s.t.}\quad w = c + ztx + \frac{11}{10}(1+\tau_0)R l_c$$

(c) IN AN OPEN CITY $c = c^*$ EVERYWHERE, SO (1) BECOMES

$$(1+\tau_0)R_0 = \frac{w - c - zt|x|}{l_c} \qquad (3)$$

AND (2) BECOMES

$$(1+\tau_1)R_1 = \frac{w - c - zt|x|}{l_c} \qquad (4)$$

(3) + (4) TOGETHER IMPLY

$$(1+\tau_0)R_0 = (1+\tau_1)R_1$$

THAT IS, THE SUM OF RENT AND TAXES DOES NOT VARY WITH THE PROPERTY TAX.

(d) NOT A GOOD IDEA. CHANGES IN PROPERTY TAX DO NOT AFFECT TOTAL EXPENDITURE ON HOUSING.

4.
(a) CONSUMPTION IN THE THREE LOCATIONS IS

$$C_1 = W - R_1 - \tau$$

$$C_2 = W - R_2 - 2\tau$$

$$C_3 = W - R_3 - 3\tau$$

THE PAYOFF IN EACH LOCATION IS $\varepsilon_i(r) C_i$, SO HOUSEHOLD r'S PROBLEM IS TO SOLVE

$$\text{MAX} \left\{ \begin{array}{l} \varepsilon_1(r)[W - R_1 - \tau], \\ \varepsilon_2(r)[W - R_2 - 2\tau], \\ \varepsilon_3(r)[W - R_3 - 3\tau] \end{array} \right\}$$

(b) USING THE THEOREM

$$S_1 = \frac{[W - R_1 - \tau]^\theta}{[W - R_1 - \tau]^\theta + [W - R_2 - 2\tau]^\theta + [W - R_3 - 3\tau]^\theta}$$

$$S_2 = \frac{[W - R_2 - 2\tau]^\theta}{[\qquad\qquad\qquad]}$$

$$S_3 = \frac{(W - R_3 - 3\tau)^\theta}{[\qquad\qquad\qquad]}$$

(c) WITH $S_1 = S_2 = S_3$, EACH OF THE THREE FRACTIONS IN (b) ARE THE SAME.

BECAUSE ALL THREE HAVE THE SAME DENOMINATOR, THE NUMERATOR MUST ALSO BE THE SAME

$$\Rightarrow \quad w - R_1 - \tau = w - R_2 - 2\tau \qquad (3)$$

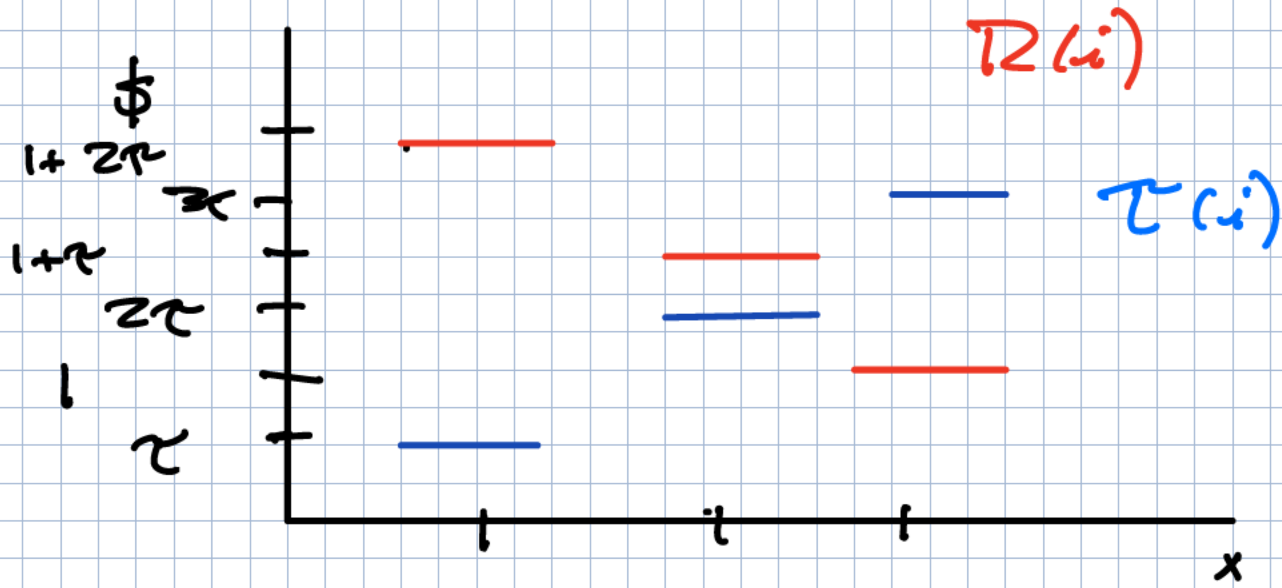
AND

$$w - R_2 - 2\tau = w - R_3 - 3\tau \qquad (4)$$

$(3) \Rightarrow \quad R_1 = R_2 + \tau$

$(4) \Rightarrow \quad R_2 = R_3 + \tau$

IF $\bar{R} = 1 = R_3$, THEN THIS MEANS THAT $(R_1, R_2, R_3) = (2\tau + 1, \tau + 1, 1)$



RENT AND COMMUTE COSTS SUM TO A CONSTANT.

5. TO ANSWER THESE QUESTIONS, FIX THE WAGE AND ASK WHAT CHANGE IN r PRESERVES UTILITY OR COSTS.

WITH w FIXED, AN INCREASE IN r MEANS UTILITY GOES UP AS A INCREASED FROM A_0 TO A_1 AND CONVERSELY.

WITH w FIXED, AN INCREASE IN r MEANS PRODUCTIVITY GOES UP, AND CONVERSELY.

THUS, IN PANEL (a) THE INCREASE IN A FROM A_0 TO A_1 DECREASES UTILITY AND PRODUCTIVITY. A IS A BAD FOR PEOPLE AND FIRMS.

IN PANEL (b) A IS GOOD FOR PEOPLE BUT BAD FOR FIRMS.