EC1410 – Spring 2022

Final Exam
May 18, 2:00-3:30pm
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You will have 110 minutes to complete this exam. No notes or books are allowed but you may use a calculator. Cell phones and any device with a wireless connection must be off. Anyone still working on their exam after time is called will be subject to an automatic 10 point penalty.

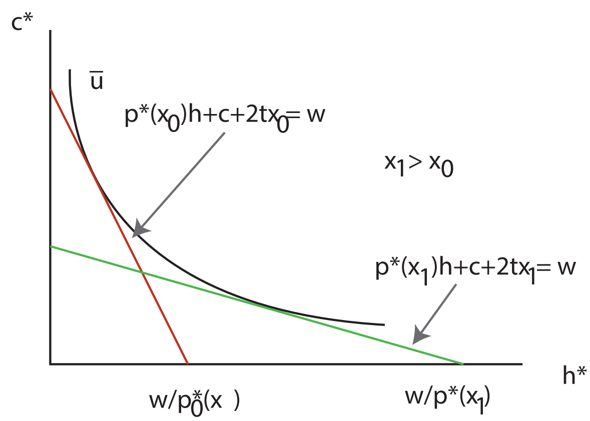
When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Total points =100/Share of total grade =40%. Points assigned to each problem are indicated in parentheses.

This exam has FOUR pages.

1. (10) Consider the household's problem in the monocentric city model with housing,

$$\max_{c,h,x} u(c,h) \text{ subject to } w = c + ph + 2tx$$

If u is such that indifference curves are concave, we can represent this problem graphically, at two different distances from the center, with the following figure.



Can you infer from this picture that the price of housing, p, is decreasing with distance? Explain briefly.

2. (20) Consider the monocentric city model with housing as discussed in lecture. Assume as well that housing production is perfectly competitive. Let $\overline{u} = 3$.

   (a) Let the household's problem be given by:

$$\max_{c,h,x} c^{1/2}h^{1/2} \text{ subject to } w = c + ph + 2tx$$

   Let $\widetilde{w} = w - 2tx$. Use the first-order condition of the household's problem with respect to h to find h^* and c^* in terms of p and $\widetilde{w}$.

(b) Use the fact that utility is $\overline{u} = 3$ and your result in part 2a to find p^* in terms of w, t, and x.

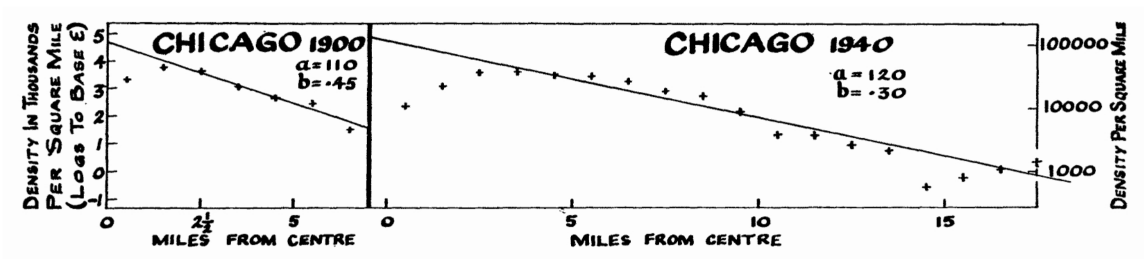
(c) Suppose housing is produced according to $h_s(S) = S^{2/3}$, where S is the capital to land ratio and h_s is housing per unit land area. Let the developer's problem be given by

$$\max_S ph_s(S) - iS - R$$

where p, i and R are the costs of housing, capital, and land, respectively. Let $i = \frac{1}{33}$.
Use the first-order condition of this problem with respect to S to solve for $h_s{}^*$ in terms of p.

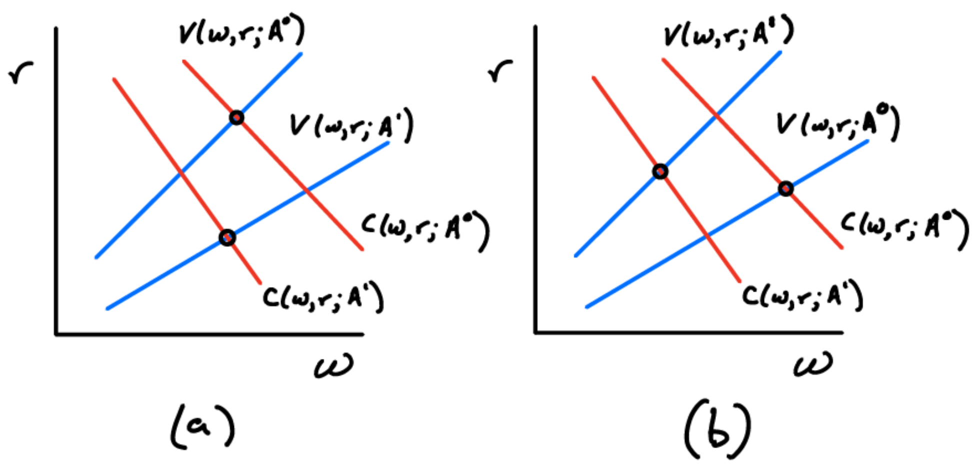
(d) Substitute using your expression for p^* from 2b to obtain an expression for population density, $\frac{h_s^*}{h^*}$, in terms of w, t and x.

(e) Consider the following figure from Clark (1951).



Why does this figure help to confirm that your answer in part 2d is correct?

3. (25) Consider the spatial equilibrium underlying the Roback Theorem. Let, $V(r,w;A)$ be the indirect utility function for city residents with arguments, w, wage, r, rent, and A, amenity. Let $C(w,r;A)$ be the unit cost function for the commercial activity in the city. Consider the following two figures showing indifference curves an iso-cost curves for two levels of an amenity, where $A_1 > A_0$.



(a) For the economy in panel (a), does the amenity increase or decrease productivity? Does it increase or decrease utility? Explain briefly.

(b) For the economy in panel (b), does the amenity increase or decrease productivity? Does it increase or decrease utility? Explain briefly.

4. (25) This problem asks you to use the Roback Theorem to calculate the importance of amenity A in real terms. Recall that the Roback Theorem states that

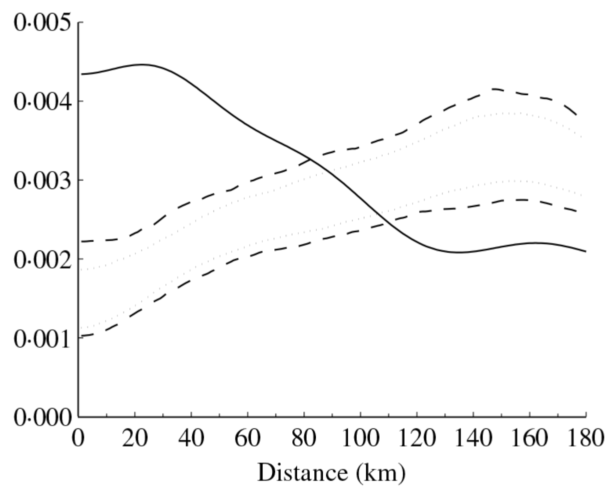
$$\begin{aligned}\frac{p_A}{w} &= \frac{\ell_c r}{w}\frac{1}{r}\frac{dr}{dA} - \frac{1}{w}\frac{dw}{dA} \\ &= \frac{\ell_c r}{w}\frac{d\ln r}{dA} - \frac{d\ln w}{dA},\end{aligned}$$

where p_A is the 'price' of the amenity, and ℓ_c is the amount of residential land consumed by a representative household in equilibrium.

Assume you have data on rents, wages, and amenity A for a cross-section of cities. That is, your data is $\{r_i, w_i, A_i\}$ for a set of cities $i = 1, ..., J$. You may also assume that housing expenditure is one-third of the city wage.

Describe how to use these data to estimate the importance of amenity A in real terms.

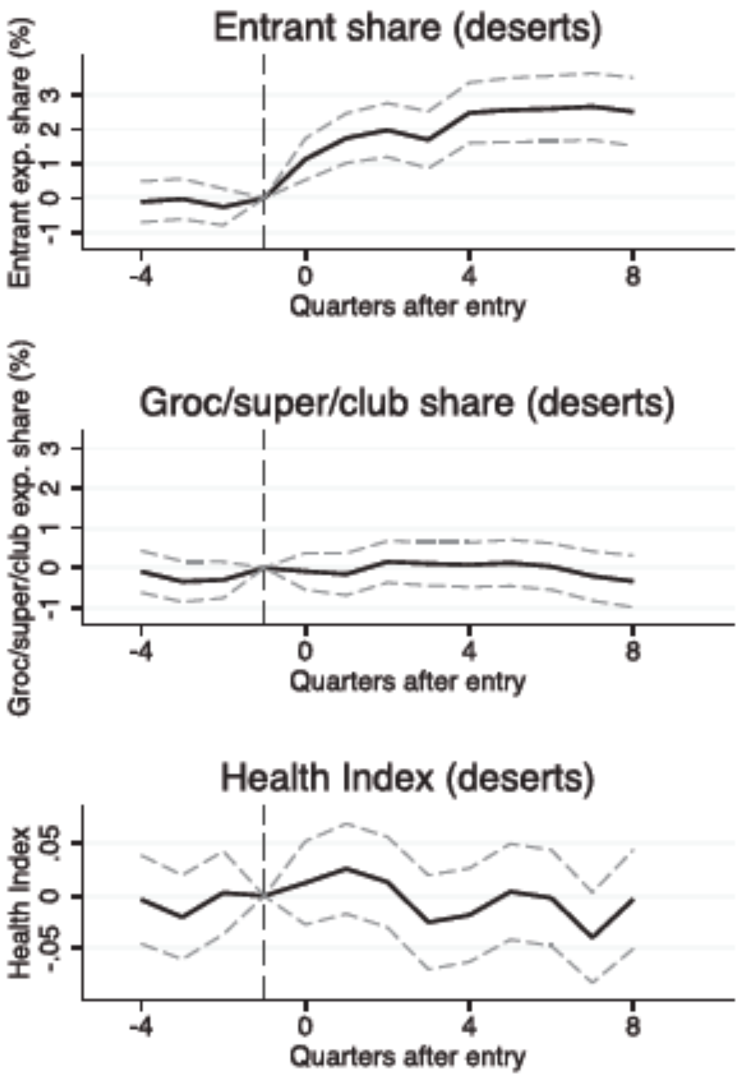
5. (10) Consider the figure describing the distribution of pairwise distances between establishments producing basic pharmaceuticals from Duranton and Overman, 2005.



(a) Basic Pharmaceuticals (SIC2441)

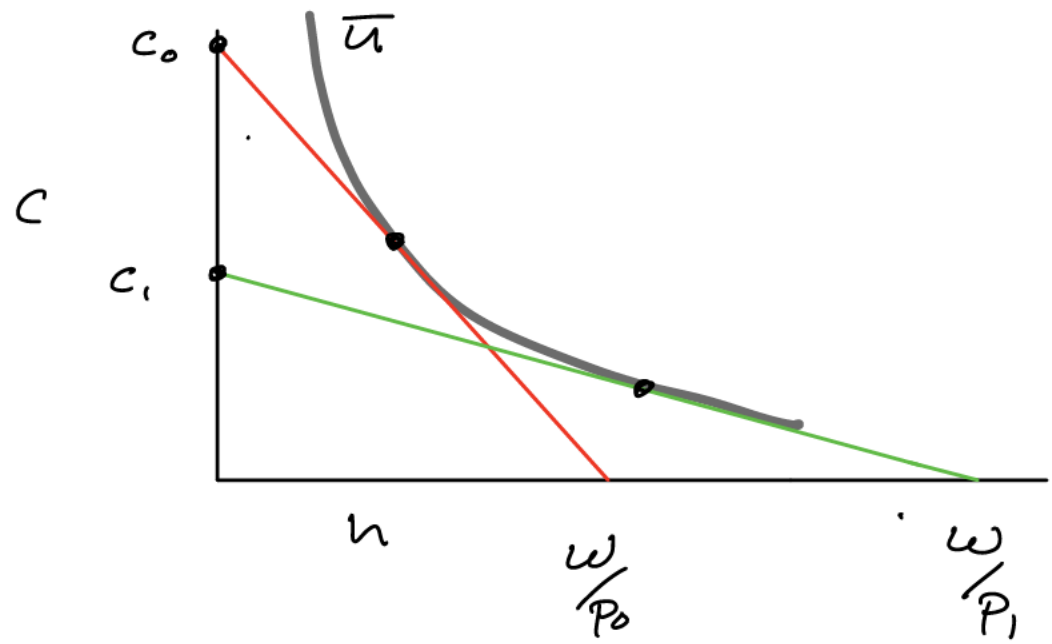
Does this figure suggest that basic pharmaceutical establishments are more or less agglomerated than we expect if they chose their locations at random? Explain briefly.

6. (10) Allcott et al. (2019) examine the relationship between diet and whether or not a person lives in a food desert. Following is one of the main figures from their analysis.



Explain briefly what this figure tells us about the relationship between food deserts and diet.

1.



- WE ARE GIVEN THAT $x_1 > x_0$. THAT IS, THE FLATTER BUDGET LINE DESCRIBES A MORE REMOTE LOCATION.

- WHEN $h=0$, WE HAVE

$$c_0 = w - 2tx_0 \quad , \quad c_1 = w - 2tx_1$$

THESE ARE y-INTERCEPTS OF THE BUDGET LINES.

- SINCE $x_1 > x_0$, WE MUST HAVE $c_1 < c_0$, AS DRAWN

- IN SPATIAL EQUILIBRIUM, HOUSEHOLDS OPTIMIZE $\Rightarrow$ BUDGET LINE IS TANGENT TO INDIF CURVE FOR OUTSIDE OPTION, $\bar{u}$.

- THIS CAN ONLY HAPPEN IF $p_1 < p_0$. THUS WE HAVE $p\downarrow$ AS $x\uparrow$

2. (a) MAX $c^{1/2} h^{1/2}$

S.T. $\tilde{\omega} = c + ph$ $\qquad \tilde{\omega} \equiv \omega - 2tx$

$\Rightarrow$ MAX $(\tilde{\omega} - ph)^{1/2} h^{1/2}$

F.O.C. $\Rightarrow \frac{1}{2}(\tilde{\omega} - ph)^{-1/2}(-p) h^{1/2} + \frac{1}{2}(\tilde{\omega} - ph)^{1/2} h^{-1/2} = 0$

$\Rightarrow -ph + (\tilde{\omega} - ph) = 0$

$\Rightarrow -2ph = -\tilde{\omega}$

$\Rightarrow h^* = \frac{\tilde{\omega}}{2p}$

$\Rightarrow c^* = \tilde{\omega} - ph^*$

$c^* = \tilde{\omega} - p\frac{\tilde{\omega}}{2p}$

$c^* = \frac{\tilde{\omega}}{2}$

(b) USING $u(c,h) = 3 \Rightarrow$

$c^{*1/2} h^{*1/2} = 3$

$\Rightarrow \left(\frac{\tilde{\omega}}{2}\right)^{1/2} \left(\frac{\tilde{\omega}}{2p}\right)^{1/2} = 3$

$\Rightarrow \frac{\tilde{\omega}}{2\sqrt{p}} = 3$

$\Rightarrow \sqrt{p} = \frac{\tilde{\omega}}{2}$

$\Rightarrow p^* = \left(\frac{\tilde{\omega}}{2}\right)^2 = \left(\frac{\omega - 2tx}{2}\right)^2$

(c) $\max_{S} \; P S^{2/3} - iS - R$

F.O.C. $\Rightarrow \quad \frac{2}{3} P S^{-1/3} - i = 0$

$\Rightarrow \quad S^{-1/3} = \frac{3}{2} \, i/P$

$\Rightarrow \quad S^* = \left(\frac{3}{2} \, i/P \right)^{-3}$

$\Rightarrow \quad S^* = \left(\frac{2}{3} \, \frac{P}{i} \right)^{3}$

SUBSTITUTING INTO $h_s(S)$

$\Rightarrow \quad h_s^*(S^*) = \left[\left(\frac{2}{3} \, \frac{P}{i} \right)^3 \right]^{2/3}$

$= \left(\frac{2}{3} \, \frac{P}{i} \right)^2$

(d) POPULATION DENSITY IS

$$\frac{h_s^*}{h^*} = \frac{\text{HOUSING}/\text{AREA}}{\text{HOUSING}/\text{PERSON}} = \frac{\text{PERSON}}{\text{AREA}}$$

$$= \frac{\left(\frac{2}{3} \, \frac{P^*}{i} \right)^2}{\frac{\tilde{\omega}}{2P^*}} = \frac{4}{9i^2} \cdot \frac{2P^{*3}}{\tilde{\omega}}$$

$$= \frac{4}{9i^2} \cdot \frac{2}{\omega - 2tx} \left[\left(\frac{\omega - 2tx}{2} \right)^2 \right]^3$$

$$= \frac{8}{9i^2} \cdot \left(\frac{w - 2tx}{2}\right)^5$$

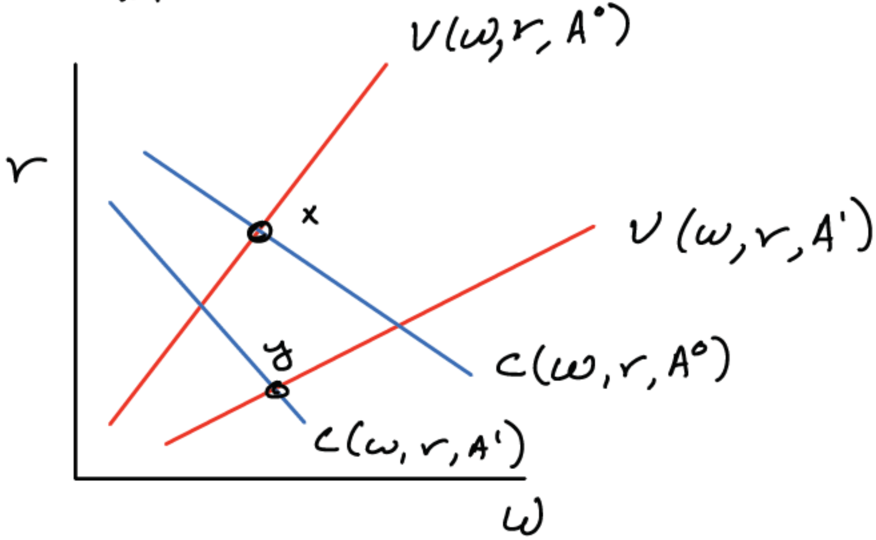
$$= \frac{1}{36i^2}\left(w - 2tx\right)^5$$

(e) • WE SEE IN (d) THAT POPULATION DECLINES WITH DISTANCE TO THE CENTER. JUST LIKE BOTH THE 1900 AND 1940 DENSITY GRADIENT.

• WE SEE THE DENSITY GRADIENT IN (e) GETS FLATTER FROM 1900 TO 1940. THIS WAS THE PERIOD WHEN THE AUTOMOBILE CAME INTO USE, SO $t \downarrow$. IN (d) THE DENSITY GRADIENT ALSO GETS FLATTER AS $t \downarrow$.

3.

(a)

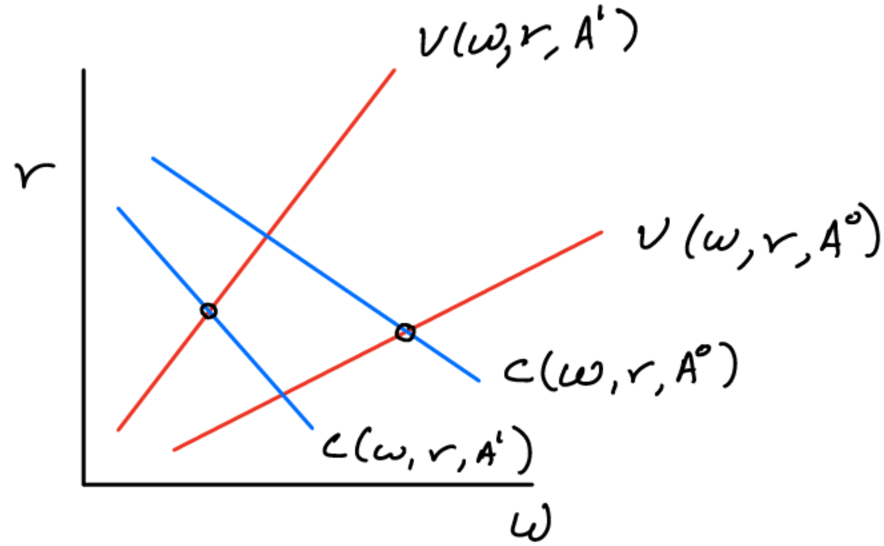


• WITH $A = A'$, EQUILIBRIUM IS AT x, WHERE $(r, w) = (r', w')$

- HOUSEHOLDS LIKE LESS RENT HOLDING W CONSTANT, SO, HOLDING UTILITY CONSTANT AND WAGE CONSTANT, HOUSEHOLDS NEED LOWER RENT TO LIVE WITH A^0 THAN A^1. SINCE $A^1 > A^0 \Rightarrow$ A IS A GOOD FOR HOUSEHOLDS.

- FIRM PRODUCE ONE UNIT AT THE SAME COST HOLDING RENT CONSTANT ONLY IF THE WAGE IS LOWER FOR A_1 THAN A_0. SINCE $A_1 > A_0 \Rightarrow A_1$ IS A BAD FOR FIRMS.

(b)



SIMILAR TO ABOVE:

- FIXING RENT, FIRMS CAN HOLD CONSTANT COSTS WHILE PAYING A HIGHER WAGE WITH A^0 THAN $A^1 \Rightarrow A^1$ IS GOOD FOR FIRMS

- FIXING RENT, H.H. HOLD UTILITY CONSTANT AT AT A LOWER RENT WITH A^1 THAN A^0.
$\Rightarrow$ NEED MORE CONSUMPTION AT AT
$\Rightarrow$ A^1 IS A BAD FOR H.H.

4. IF WE PERFORM THE REGRESSIONS

$$\ln r = B_0 + B_1 A + \varepsilon$$

AND

$$\ln w = C_0 + C_1 A + \eta$$

THEN $$\frac{d \ln r}{dA} = B_1$$

AND $$\frac{d \ln w}{dA} = C_1$$

WE CAN DO THESE REGRESSIONS WITH GIVEN DATA.

WE ARE ALSO GIVEN THAT $\frac{l_c r}{w} = 1/3$

THUS WE HAVE

$$\frac{P_A}{w} = \frac{1}{3} \cdot B_1 - C_1$$

THAT IS, WE CAN ESTIMATE THE REAL PRICE OF A FROM THE GIVEN DATA.

6. • THE SOLID LINE IN THE FIGURE IS THE OBSERVED DISTRIBUTION OF PAIRWISE DISTANCES FOR THIS INDUSTRY.

• THE TWO DASHED LINES BOUND THE AREA WHERE WE WOULD EXPECT THIS CURVE TO LIE IF FIRMS LOCATED AT RANDOM.

• THE GRAPH SHOWS TOO MANY PAIRS OF FIRMS CLOSE TOGETHER FOR RANDOM CHOICE

⟹ FIRMS ARE MORE AGGLOMERATED THAN RANDOM

7. TOP: NEW BIG GROCERY ENTRY DRAWS ABOUT 3% OF ALL FOOD EXPENDITURE FROM FOOD DESERT RESIDENTS

MIDDLE: BIG GROCERY STORE SHARE OF ALL FOOD EXPENDITURE FROM FOOD DESERT RESIDENTS DOES NOT CHANGE WITH ENTRY OF NEW GROCERY

BOTTOM: DIET OF FOOD DESERT RESIDENTS DOES NOT CHANGE WITH ENTRY OF STORE.

⟹ ADDING GROCERIES TO FOOD DESERTS DOES NOT CHANGE DIETS OF RESIDENTS.