EC1340-Fall 2023
Problem Set 5

(Updated 18 August 2023)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

In the case of empirical exercises, your goal should be to provide enough information to allow a reader to replicate your answer. This requires a description of data and data sources as well as a description of your analysis of the data.

Answers which do not achieve these goals will not be awarded full credit.

Problems

1. Let $t = 0,1,2,...$ index years. Suppose that one ton of CO_2 emissions today causes 0\$ of damage for $t < 100$ and 50\$ of damage for $t \geq 100$. Let M denote the amount spent on mitigation at $t = 0$. If the interest rate is r how much will a planner who maximizes the discount present value of consumption be willing to spend on abatement to reduce future damage to zero.

2. Prove that $\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t = \frac{1}{r}$.

3. One problem with discounting is that it counts future benefits very little after we get more than a few years ahead. To see this, conduct the following three exercises:

   (a) Write the expression for the present value of a one hundred dollar payoff 100 years from now as a function of the interest rate.

   (b) Policy makers actually use discounting when they are trying to make decisions. Commonly used discount rates range between 3% and 10%. Stern uses 1.3% and Nordhaus 5.5%. What is the present value of our 100\$ payoff 100 years from now for $r = 1.3\%$, 5.5% and 10%.

4. The preceding problem illustrates the problems with discounting. Here is the problem with not discounting. The problem with discounting is that it seems to give "too little" weight to the future. Suppose instead that we give all future years exactly the same weight as the current year. Thus, given streams of payoffs $(c_t)_{t=0}^{\infty}$ we will rank them according to their sum. There are two problems with this method.

   (a) Let $c_t = \begin{cases} -1000 & if \quad t = 0 \\ 0.00001 & if \quad t \geq 1 \end{cases}$, and let $c'_t = 0$ for all t.

   If I do not discount future benefits, which policy should I choose?

   (b) Consider two policies
   $$c_t = \begin{cases} 1 & if \quad t \text{ odd and greater than 1} \\ 2 & if \quad t \text{ even} \\ 1.1 & if \quad t = 1 \end{cases}$$
   and
   $$c'_t = \begin{cases} 2 & if \quad t \text{ odd} \\ 1 & if \quad t \text{ even} \end{cases}.$$

   If you use the undiscounted sum of payoffs can you rank these two policies?