

# EC1340-Fall 2021 Problem Set 2 solutions

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1. (a)

$$T_1 = (2,0; \frac{1}{2}, \frac{1}{2}), \quad E(T_1) = 1, \quad V(T_1) = 1$$

$$T_2 = (6,4; \frac{1}{2}, \frac{1}{2}), \quad E(T_2) = 5, \quad V(T_2) = 1$$

(b)

$$T^* = \left( \left( 2,0; \frac{1}{2}, \frac{1}{2} \right), \left( 6,4; \frac{1}{2}, \frac{1}{2} \right); \frac{1}{2}, \frac{1}{2} \right)$$

$$= \left( 2,0,6,4; \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right)$$

$$E(T^*) = 3$$

$$V(T^*) = 5$$

(c) This one is a little harder because you need to figure out the probabilities of realizations of both lotteries in order to evaluate the mean. The following table shows how:

p	$T_1$	$T_2$	$T^{**}$
$\frac{1}{4}$	2	6	4
$\frac{1}{4}$	2	4	3
$\frac{1}{4}$	0	6	3
$\frac{1}{4}$	0	4	2

So we have  $T^{**} = \left( 4,3,3,2; \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right)$

It follows that  $E(T^{**}) = 3$  and  $V(T^{**}) = 1/2$

(d) In part (b) we randomized over two models, and in part (c) we averaged over the same two models. In both cases we got the same expected value, but in the second we have lower variance. This suggests that both rules will lead to the same temperature predictions on average, but that model averaging reduces uncertainty. This suggests that model averaging is probably better.

(e) The IPCC either failed to predict the warming hiatus from 1998-2015, or alternatively, they failed to estimate how uncertain their estimates were.

Here is an example of how this might have occurred. Consider a climate scientist at the IPCC who is particularly attached to his or her own climate model, say  $T_1$  in the problem above. Because of this attachment, the scientist fails to consider the fact that there is an alternative model  $T_2$  and that he or she has no basis for choosing between them. In this case, the analyst reports the smaller prediction of model  $T_1$ , when the true lottery/model is actually

$T^*$ , and hence under reports the uncertainty of their estimates. This makes it likely that an event will occur, like the warming hiatus, that is outside the range of events that he or she predicted could happen.

A better strategy is to acknowledge model uncertainty and to devise a strategy to deal with it, for example, model averaging.

2. Each of the following numbers is an important constant that is either part of the BDICE model, or it helps to understand some other important aspect of the climate change problem. For each number, give the appropriate units and a brief description. For example, '240' would be something like '240 W/m<sup>2</sup>. This is the baseline amount of energy that the earth receives from the sun.'
- (a)  $\frac{44}{12}$  CO<sub>2</sub>/C. Multiplying mass of Carbon by this constant gives mass of Carbon Dioxide.
- (b)  $\frac{3}{280}$  degrees Celsius of warming over 100 years per ppm of atmospheric concentration of CO<sub>2</sub>. This is 'Nordaus' rule of thumb': doubling CO<sub>2</sub> from preindustrial levels (280ppm) to 560ppm results in 3 degrees of warming by 2100.
- (c) 55% this is fraction of each ton of CO<sub>2</sub> emissions that stays in the atmosphere to contribute to the atmospheric concentration of CO<sub>2</sub>.
- (d) 2.12 Gt/ppm. This is a physical constant. It is the weight of Carbon required to increase atmospheric concentration of Carbon by 1ppm.
- (e) 0.17 kgC/\$. This is the (approximate) average amount of carbon emissions per dollar of economic output. It is calculated by dividing world emissions by world GDP.
- (f) 0.77 is the fraction of all CO<sub>2e</sub> in the atmosphere that is actually CO<sub>2</sub>.
- (g)  $77 \times 10^{12}$  dollars is approximate world GDP in 2013.
- (h)  $13 \times 10^9$  tons Carbon (not CO<sub>2</sub>) is approximate world emissions in 2013.