University of Toronto
Faculty of Arts and Science
December Examinations 2012
ECO313H1F — Matthew A. Turner
Duration - 2 hours

Examination Aids: No notes or books are allowed, but you may use a calculator.

When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Point counts of individual problems are indicated in parentheses. Total points =100.

1. This problem asks you to identify the optimal type of regulatory instrument—price or quantity—in an environment where the planner is uncertain about the firm's costs.

   Let

   $$\begin{aligned} B(y) &= 3y - \frac{1}{2}y^2 \\ C(y) &= \eta y + \frac{1}{2}y^2, \end{aligned}$$

   where η is a random variable that affects the firm's costs. Define η as follows:

   $$\eta = \begin{cases} 1 & p = \frac{1}{2} \\ 0 & p = \frac{1}{2} \end{cases}.$$

   The planner must choose between optimal quantity regulation, y^* and optimal price regulation $\widehat{p}$.

   (a) (15) Establish analytically whether quantity or price regulation is socially optimal.

   (b) (10) Draw a graph that illustrates: Marginal cost function (in each state of the world), the marginal benefit function, optimal price regulation $\widehat{p}$, firm response to price regulation $\widehat{y}(\widehat{p})$ and optimal quantity regulation y^*. Also illustrate the deadweight loss from each instrument in each state of the world.

2. Suppose two firms produce steel and have cost functions $c_1(y_1) = 3y_1^2$, and $c_2(y_2) = 2y_2^2$. The market price of steel is $p = 5$. Steel is jointly produced with smoke, and a planner has determined that social welfare is maximized when only one unit of steel is produced.

   (a) (5) Suppose the planner regulates production by issuing each firm non-tradable quota to produce half a unit of steel. What are the total costs to produce the one unit of steel? What are firms' profits?

   (b) (10) Now suppose that firms are able to trade their initial half units of quota at a market price p_Q. Find the price at which the quota market clears.

   (c) (10) What is the most that the government can charge firms for their initial allocation of tradable quota, and still expect firms to prefer tradable quota to non-tradable quota?

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3. 'Measuring temperature: the 170 year Thermometer-based record' suggests that we ought to be sceptical of measured temperature data showing an increase in temperature because individual instruments are subject to measurement error. While there are problems with the measured temperature record, this turns out not to be one of the important ones. This exercise asks you to work out the math to demonstrate this.

   Suppose we have two thermometers which each produce a reading T_i for $i = 1,2$. For each reading, this error consists of the true temperature and an error e_i. e_i is random and takes the values x and $-x$ with equal probability. Let T denote the average recorded temperature across the two locations, that is, $T = (T_1 + T_2)/2$. Let $E(T_i)$ denote the expected value of T_i, $var(T_i)$ denote variance and $sd(T_i) = \sqrt{var(T_i)}$ the standard deviation. Assume the the errors are independent of each other and the true temperature is the same for both thermometers/locations.

   (a) (5) Calculate $E(T_i)$.
   (b) (5) Calculate $SD(T_i)$.
   (c) (5) Calculate the expected or average value of T.
   (d) (10) Calculate the average error (or standard deviation) for T.

4. Current world income is about 63 trillion dollars. Consider the two following growth paths.

   - World income grows at 1.5% a year forever, but in 100 years is subject to a 5% decrease. This corresponds (approximately) to the case Nordhaus analyzes: after it warms up, productivity drops, but growth continues largely unharmed. The discounted present value of world income on this path is

$$W_1 = \sum_{t=0}^{99} \left(\frac{1}{1+r}\right)^t (1.015)^t Y_0 + \left(\frac{1}{1+r}\right)^{100} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (1.015)^t [0.95(1.015)^{100} Y_0]$$

   - World income grows at 1.4% forever. This path is the mitigation path. We solve the problem of warming at time 0 and but experience slightly lower growth. The discounted present value of world income on this path is

$$W_2 = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (1.014)^t Y_0$$

   The value of solution to the mitigation problem in W_1 is the difference between W_2 and W_1.

   (a) (15) Evaluate $W_2 - W_1$ for $r = 2\%$ and $r = 5.0\%$.
   (b) (10) It is (probably) not correct to use the same discount rates to evaluate the two scenarios. Why? Explain briefly.

Total Marks = (100)
Total Pages = (2).

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Solutions

1. (a) First, find the optimal quantity regulation y^*,

$$\begin{aligned} &\max E(B(y) - C(y)) \\ =&\max E(3y - \frac{1}{2}y^2 - \eta y - \frac{1}{2}y^2) \\ =&\max 3y - y^2 - E(\eta)y \\ =&\max 3y - y^2 - \frac{1}{2}y \\ =&\max \frac{5}{2}y - y^2 \end{aligned}$$

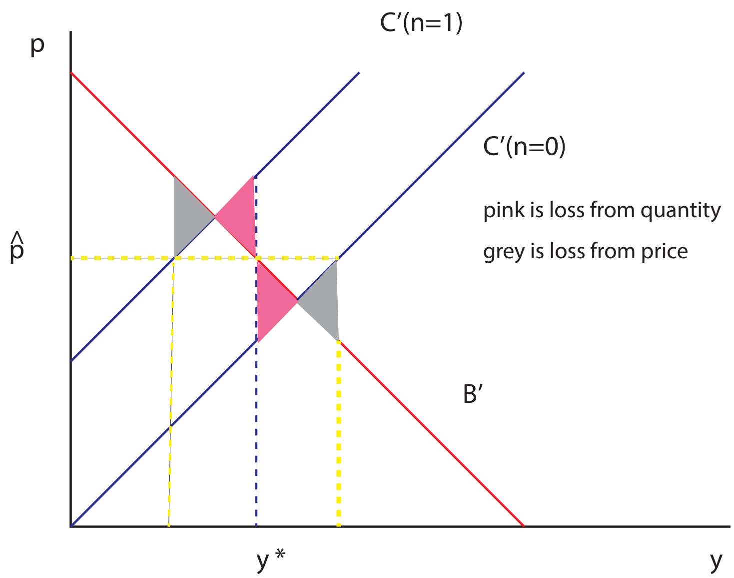
The first order condition is $2y = 5/2$ so $y^* = 5/4$
Subsituting y^* into the last of the set of equations above, we have that $W(y^*) = \frac{5}{2}(\frac{5}{4}) - (\frac{5}{4})^2 = 75/16$.
Now find the firm's response to price regulation, $\hat{y}(p)$. The firm solves

$$\begin{aligned} &\max py - C(y) \\ =&\max py - \eta y - \frac{1}{2}y^2) \end{aligned}$$

The first order condition here is $y = p - \eta$, so we have $\hat{y}(p) = p - \eta$. Note that the firm observes its costs and so does not make decisions under uncertainty.
Next, use the firm's response function to find the best price regualtion, $\hat{p}$. In this case, the planner solves,

$$\begin{aligned} &\max E(B(\hat{y}(p)) - C(\hat{y}(p))) \\ =&\max E(3(p-\eta) - \frac{1}{2}(p-\eta)^2 - \eta(p-\eta) - \frac{1}{2}(p-\eta)^2) \\ =&\max 3p - p^2 - 3E(\eta) - pE(\eta) \\ =&\max \frac{7}{2}p - \frac{3}{2} - p^2 \end{aligned}$$

The first order condition is $\frac{7}{2} = 2p$ so we have that $\hat{p} = 7/4$.
Substituting $\hat{p} = 7/4$ into the last of the equations above, we have that $W(\hat{p}) = \frac{7}{2}(\frac{7}{4}) - \frac{3}{2} - (\frac{7}{4})^2 = 25/16$.
Thus, $W(\hat{p} < W(y^*)$ and quantity regulation is optimal.



(b)

2. (a) If each firm produces 1/2 a unit of steel then $c_1(\frac{1}{2}) + c_2(\frac{1}{2}) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$ Profits are $\pi_1 = 7/4$ and $\pi_2 = 2$

(b) Firm 1 solves

$$\max (5 - p_Q) y_1 - 3y_1^2 + \frac{1}{2} p_Q$$
$$\Longrightarrow 5 - p_Q = 6y_1$$
$$\Longrightarrow y_1(p_Q) = \frac{5 - p_Q}{6}$$

Firm 2 solves

$$\max (5 - p_Q) y_2 - 2y_2^2 + \frac{1}{2} p_Q$$
$$\Longrightarrow 5 - p_Q = 4y_2$$
$$\Longrightarrow y_2(p_Q) = \frac{5 - p_Q}{4}$$

The quota market clears when

$$y_1(p_Q) + y_2(p_Q) = 1$$
$$\Longrightarrow p_Q = \frac{13}{5}$$

(c) With $p_Q = \frac{13}{5}$ we have

$$\pi_1 = (5 - \frac{13}{5}) \frac{5 - p_Q}{6} - 3(\frac{5 - p_Q}{6})^2 + \frac{1}{2}\frac{13}{5}$$
$$= 89/50$$

and

$$\pi_2 = (5 - \frac{13}{5}) \frac{5 - p_Q}{4} - 2(\frac{5 - p_Q}{4})^2 + \frac{1}{2}\frac{13}{5}$$
$$= 2\frac{1}{50}$$

Firms prefer tradable or non-tradable quota according to which gives them higher profits. In part (a) we found profits under non-tradable quota, in (b) under tradable quota. We find that Firm 1 make 1/50 and firm 2 3/100 more dollars under tradable than non-tradable quota, so neither would be willing to pay more than that to switch from non-tradable to tradable quota.

3. (a) $T_i = (\frac{1}{2},\frac{1}{2}; t+x, t-x)$ so that $E(T_i) = \frac{1}{2}(t+x) + \frac{1}{2}(t-x) = t$

   (b) $var(T_i) = \frac{1}{2}[(t+x) - E(T_i)]^2 + \frac{1}{2}[(t-x) - E(T_i)]^2 = x^2$. So $SD(T_i) = \sqrt{var(T_i)} = x$.

   (c) $T = (\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}; t+x, t, t, t-x)$ so that $E(T) = \frac{1}{4}(t+x) + \frac{1}{2}(x) + \frac{1}{4}(t-x) = t$

   (d) $var(T) = \frac{1}{4}[(t+x) - E(T)]^2 + \frac{1}{2}[(t) - E(T)]^2 + \frac{1}{4}[(t-x) - E(T)]^2 = \frac{1}{2}x^2$. So $SD(T_i) = \sqrt{var(T)} = x/\sqrt{2}$.

4. (a) To make this problem easier to handle, define $\delta_1^1 = \frac{1.014}{1.02} = 0.994$, $\delta_1^2 = \frac{1.014}{1.05} = 0.966$, $\delta_2^1 = \frac{1.015}{1.02} = 0.995$ and $\delta_2^2 = \frac{1.015}{1.05} = 0.967$. With this notation in place, we have

$$\begin{aligned} W_1 &= \sum_{t=0}^{99} \delta_1^t Y_0 + \delta_1^{100} \sum_{t=0}^{\infty} \delta_1^t (0.95 Y_0) \\ &= \frac{1-\delta_1^{100}}{1-\delta_1} Y_0 + \delta_1^{100} \frac{1}{1-\delta_1} (0.95 Y_0) \\ W_2 &= \sum_{t=0}^{\infty} \delta_1^t Y_0 \\ &= \frac{1}{1-\delta_1} Y_0 \end{aligned}$$

   Evaluate $W_1 - W_2$ using the formulas above, to get

$$\begin{aligned} W_2 - W_1|_{r=.02} &= -27.32 Y_0 \\ W_2 - W_1|_{r=.05} &= -.84 Y_0 \end{aligned}$$

   (b) Recall from our derivation of the discount rate that the discount rate depends on g, the growth rate of consumption. In the two scenarios above, this rate is not the same, so we should probably not expect the discount rate to be the same across the two cases.