EC1340 Topic #9

Optimal warming policy

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The problem of stabilizing CO_2

Consider the problem we started with:

- World emissions of CO_2e in 2019(2022 IPCC report) were about 59 Gt. Stabilizing atmospheric concentrations (not temp) requires cutting this to about 25Gt.
- There are about 8 bn people in the world as of 2022. Stabilization requires reducing emissions to 25Gt/8bn ≈ 3.0Gt CO_2e $\approx$ 1t C emissions per person.
- 2019 per capita CO_2e /incomes are about: US, 18.2t/69,000\$; China is 11.0/12,500\$; India is 2.3/2300\$.
- The US needs between a 50% and 80% reduction if the world is to reach this target.

Stern/Gore/Hansen/Nordhaus disagree about the rate at which we should approach this goal, but not about the goal.

Optimal mitigation policy

To tackle this problem, we wrote it as the BDICE model

$$\max_{I,M} \frac{c_1^{1-\alpha}}{1-\alpha} + \frac{1}{1+\rho}\frac{c_2^{1-\alpha}}{1-\alpha}$$

$$\text{s.t. } W = c_1 + I + M$$

$$c_2 = (1+r)I - \gamma(T_2 - T_1)I$$

$$E = (1 - \rho_4\frac{M}{W})(\rho_5(c_1 + I))$$

$$P_2 = \rho_0 E + P_1$$

$$T_2 = \rho_1(P_2 - P_1) + T_1$$

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The BDICE model organizes the main ideas, but leaves out some important things:

- Timing. When should we invest in education/factories and when in mitigation? To fix, use many time periods instead of two.

- Population growth. To allow this, Nordhaus uses exogenous population growth to match predictions. Endogenous would be better.

- Technical progress. Exogenous versus endogenous? CO_2 reducing versus not? Both exogenous in Nordhaus.

- Non-linearities in climate response to CO_2 and in carbon cycle. Nordhaus uses simple models calibrated to reproduce complicated models.

- Multiple countries. Not in our books, but treated in later work (RICE 2010)

DICE model (partial)

Here are variable definitions for the DICE model

$$Q(t) = \text{total output (gdp) at } t$$
$$L(t) = \text{population at } t$$
$$C(t) = \text{aggregate consumption at } t$$
$$c(t) = C(t)/L(t) \text{ per capita consumption}$$
$$I(t) = \text{total savings at } t$$
$$K(t) = \text{capital at } t$$
$$E(t) = \text{emissions at } t$$
$$A(t) = \text{level of technology at } t$$
$$\Lambda(t) = \text{cost of mitigation as \% of } Q(t)$$
$$1 - \Omega(t) = \text{loss of output from climate at } t$$

Here are main equations for the DICE model

$$W = \sum_{t=0}^{24} L(t) \frac{c(t)^{1-\alpha}}{1-\alpha} \left(\frac{1}{1+\rho} \right)^t \tag{1}$$

$$Q(t) = \Omega(t) \left[1 - \Lambda(t) \right] A(t) K(t)^{\gamma} L(t)^{1-\gamma} \tag{2}$$

$$Q(t) = C(t) + I(t) \tag{3}$$

$$K(t) = I(t) + (1 - \delta_K) K(t-1) \tag{4}$$

$$E(t) = \sigma(t) \left[1 - \mu(t) \right] A(t) K(t)^{\gamma} L(t)^{1-\gamma} \tag{5}$$

$$\Lambda(t) = \pi(t) \theta_1(t) \mu(t)^{\theta_2} \tag{6}$$

plus a description of the way climate, carbon, population and technology evolve.

What is all of this stuff!

- Equation 1 – utility function

$$W = \sum_{t=0}^{24} L(t) \frac{c(t)^{1-\alpha}}{1-\alpha} \left(\frac{1}{1+\rho}\right)^t$$

$$= L(0) \frac{c(0)^{1-\alpha}}{1-\alpha} +$$

$$L(1) \frac{c(1)^{1-\alpha}}{1-\alpha} + \left(\frac{1}{1+\rho}\right)^1$$

$$L(2) \frac{c(2)^{1-\alpha}}{1-\alpha} + \left(\frac{1}{1+\rho}\right)^2 + ...$$

This is a generalization of the CRRA utility function to many periods AND weights each period by population.

- Equation 2 – production

$$Q(t) = \Omega(t) \left[1 - \Lambda(t)\right] A(t) K(t)^{\gamma} L(t)^{1-\gamma}$$

$Q(t) = $ total output (gdp) at t

$L(t) = $ population at t

$K(t) = $ capital at t

$A(t) = $ level of technology at t

$\Lambda(t) = $ cost of mitigation as % of $Q(t)$

$1 - \Omega(t) = $ loss of output from climate at t

- Equation 3 – Budget constraint at *t*

$$Q(t) = C(t) + I(t)$$

$$C(t) = \text{aggregate consumption at } t$$
$$I(t) = \text{total savings at } t$$

- Equation 4 – Evolution of capital

$$K(t) = I(t) + (1 - \delta_K)K(t - 1)$$

$$I(t) = \text{total savings at } t$$
$$K(t) = \text{capital at } t$$

δ_K is 'depreciation rate'.

- Equation 5 – Emissions

$$E(t) = \sigma(t) \left[1 - \mu(t)\right] A(t) K(t)^{\gamma} L(t)^{1-\gamma}$$

$L(t) = $ population at t

$K(t) = $ capital at t

$E(t) = $ emissions at t

$A(t) = $ level of technology at t

$\mu(t) = $ share of mitigation at (policy variable) t

$\sigma(t) = $ Gt Carbon per unit output at t

The RHS of this basically eq 2 (output) $\times \sigma(t)$.

- Equation 6 – mitigation cost function

$$\Lambda(t) = \pi(t)\theta_1(t)\mu(t)^{\theta_2}$$

$\theta_1(t), \theta_2(t) =$ mitigation cost parameters at t

$\pi(t) =$ participation cost markup at t

$\Lambda(t) =$ cost of mitigation as share of $Q(t)$

The 'low cost backstop' policy involves modifying Λ so each ton of mitigation is 5\$ (Nordhaus, p77). This is Nordhaus' stylized description of geo-engineering.

- All together again,

$$W = \sum_{t=0}^{24} L(t) \frac{c(t)^{1-\alpha}}{1-\alpha} \left(\frac{1}{1+\rho} \right)^t$$

$$Q(t) = \Omega(t) \left[1 - \Lambda(t) \right] A(t) K(t)^\gamma L(t)^{1-\gamma}$$

$$Q(t) = C(t) + I(t)$$

$$K(t) = I(t) + (1 - \delta_K) K(t-1)$$

$$E(t) = \sigma(t) \left[1 - \mu(t) \right] A(t) K(t)^\gamma L(t)^{1-\gamma}$$

$$\Lambda(t) = \pi(t) \theta_1(t) \mu(t)^{\theta_2}$$

A solution is $((\mu(0), c(0)), ..., (\mu(24), c(24))$ with $\frac{\partial W}{\partial c(t)} = \frac{\partial W}{\partial c(t')}$ for all t, t' (more or less).

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This is a nice, careful statement of the problem. It is useful to have a single summary measure. For this purpose, people use the 'Social Cost of Carbon'

$$\frac{\frac{\partial W}{\partial E_t}}{\frac{\partial W}{\partial C_0}} \approx \frac{\partial W}{\partial E_t}$$
$$\equiv SCC_t$$

Carefully, this is the social cost of CO_2 emissions at t in terms of consumption at $t = 0$.

This is the number that people talk about as the 'optimal' carbon tax. We'll talk about why later. This is a partial description of the

DICE model. Things I've skipped:

- How climate is affected by CO_2 .
- How emissions affect atmospheric CO_2 .
- How the stock of CO_2 evolves in the atmosphere. It's all there, and looks like what we've talked about in class.
- The rate of return to capital, r, doesn't occur explicitly in this model. Instead we have, a social rate of time preference ρ, 'inequality aversion', here it's α. Consumption growth is hidden. It's the rate of change of c_t. The rate of return to capital is $\alpha g + \rho = r$.

Using the DICE model, for any mitigation path we can find

- gdp at t
- emissions at t
- carbon concentration at t
- climate at t
- carbon price at t

We can also look at the mitigation paths, $\mu(t)$, that accomplish different goals. For example:

1. $\mu(t) = 0$
2. $\mu(t)$ maximizes W – the optimal policy
3. $\mu(t)$ maximizes W and $CO_2 \leq$ cap
4. $\mu(t)$ maximizes W and temperature $\leq$ cap
5. $\mu(t)$ approximates Kyoto – a fraction of countries restrict emissions to 1990 levels
6. Stern review – strict cap on CO_2
7. Gore plan – strict cap on CO_2
8. low cost backstop

The 'value' of a policy 2-8 is the difference between W under that policy and policy (1).

Value of different polices relative to 'do nothing for 250 years':
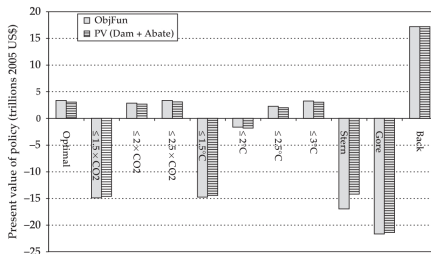
Figure 5-1. Present value of alternative policies. The difference in the present value of a policy relative to the baseline under two measures. The first bar is the value of the objective function in 2005 dollars (ObjFun), and the second is the present value of the sum of abatement and damages in the same units [PV (Dam +Abate)]. The policies are shown in Table 4-1. The baseline is omitted because it has zero present-value difference.

Note: World annual income was about 50T in 2005. This graph says global warming is a small problem! Why? Discounting and small damages.
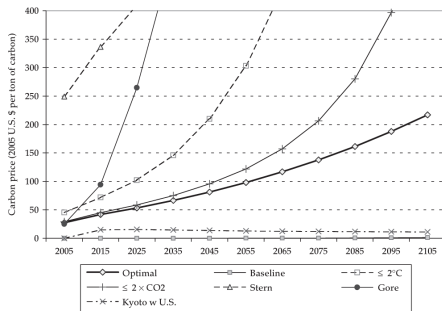
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Figure 5-4. Carbon prices under different policies. The globally averaged carbon price under different policies over the next century. Note the upward tilt of the strategies. These prices are per ton of carbon; for prices per ton of CO_2, divide by 3.67.

- This is Social Cost of Carbon at time t of emissions at time t (in terms of consumption at time t?).

- Note the conflation of 'Social Cost of Carbon' with 'Carbon Price'. This may seem puzzling, but it is OK – more later.

- This is SCC of c . To get SCC of CO_2 , multiply by $12/44$.
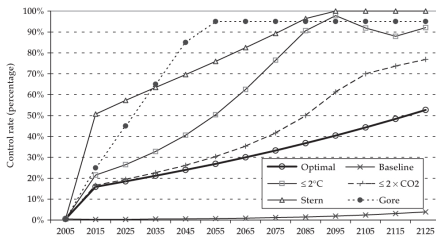
Figure 5-5. Emissions-control rates under different policies. The global emissions-control rate for CO_2 under different policies over the next century. Note the upward-tilted ramp of the strategies.
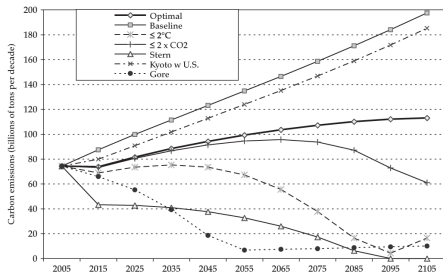
Figure 5-6. Global emissions of industrial CO_2 per decade under different policies. The global emissions of industrial CO_2 under different policies over the next century. The figure for 2005 is the actual value.

- Emissions continue to increase on Nordhaus' optimal plan, but slowly.
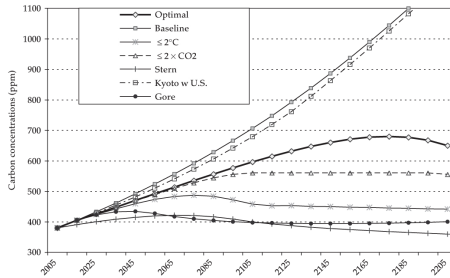- Label on y-axis is c . In note, it is CO_2 . Which is right? (y-axis, why?)

Figure 5-7. Atmospheric CO_2 concentrations under different policies. The atmospheric concentrations of CO_2 under different policies over the next century. The figure for 2005 is the actual value.

Concentrations start to fall under optimal plan, even with rising emissions. Why? (Carbon cycle, and 200 years vs 100 years on x-axis!)
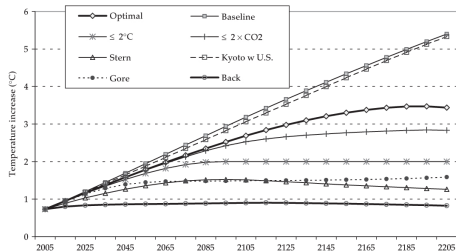
Figure 5-8. Projected global mean temperature change under different policies. Increases are relative to the 1900 average.
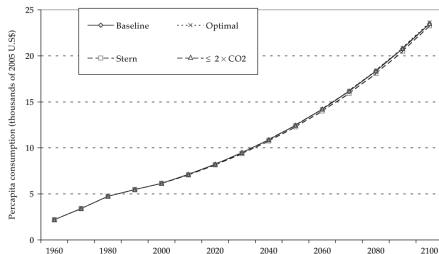
Figure 5-9. Per capita consumption, major runs. The trend of per capita consumption is strongly rising in the DICE-2007 model projections. Also, the levels of consumption are virtually indistinguishable among the different policies.

This figure also shows that global warming is a 'small problem'. The really important thing in this model is economic growth.

Conclusion from DICE model

Using Nordhaus' model we find that the optimal policy calls for a modest initial price of CO_2 which rises over time.
This occurs because, at least for the next 50 years, it looks like we'll get rich much faster if we invest in capital than in mitigation. After 50 years, we'll be able to afford much more rapid mitigation, and it won't hurt as much because we're starting from a much higher income level.

Issues with DICE

- DICE uses the following assumptions
  - r=5.5%
  - Damages, more-or-less, from developed world agriculture. No fires, hurricanes or plagues, etc.
  - No 'tipping points' (c.f. Oreske and Owens).
  - No growth effects.
  - No uncertainty.

  DICE uses the most conservative defensible assumptions.
  This still leads us to conclude that we need a carbon tax. We can debate whether the Carbon tax should be 40$ per ton, or 400$. That it should be positive seems settled.
- We've talked about the quality of the underlying data. There is lots of room for improvement
- What about uncertainty?
- What about 'stewardship' or sustainability?
- What about differences between countries? (RICE model, cf. Waldinger (2023))

... and DICE 2008 is pretty old

The DICE model was based on science and data that date from around 2005. There have been some updates to data and parameters since then. Nordhaus and Barrage 2023, is an update. Main changes are:

- $r = 3.5\%$ down from 5.5%
- Damage function reflects slightly higher damages.
- Include all CO_2 emissions, not just fossil.

The resulting new estimates are presented on the following slides. New picture on the right, old one on the left. Note that the axes might not be the same.
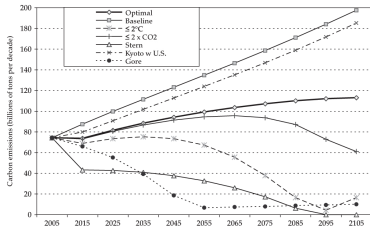
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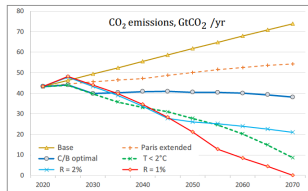
Figure 1. Results for CO_2 emissions in different scenarios

1. y-axis is different. Old is Fossil CO_2 per decade. New is all CO_2 per year. (Does this work out? $\frac{80}{10} \times \frac{44}{12} \approx 30$)

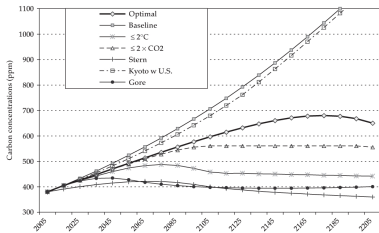2. Old optimal path called for slow increase. Current calls for freezing emissions at current level.

Figure 5-7. Atmospheric CO₂ concentrations under different policies. The atmospheric concentrations of CO₂ under different policies over the next century. The figure for 2005 is the actual value.
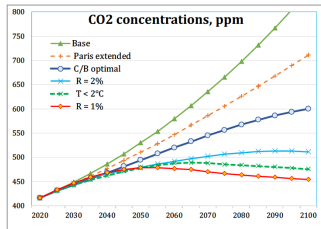

Figure 2. CO₂ concentrations, ppm

1. x-axis scale is different.
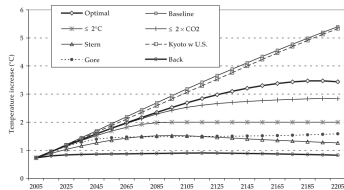2. Old and new both call for 600ppm by 2100.

Figure 5-8. Projected global mean temperature change under different policies. Increases are relative to the 1900 average.
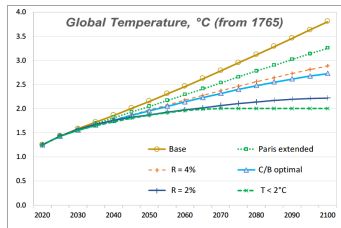
Figure 3. Global temperature increases under different scenarios

1. x-axis scale is different.
2. Old and new both call for about 2.5-3.0$C°$ by 2100.

Figure 5-5. Emissions-control rates under different policies. The global emissions-control rate for CO_2 under different policies over the next century. Note the upward-tilted ramp of the strategies.
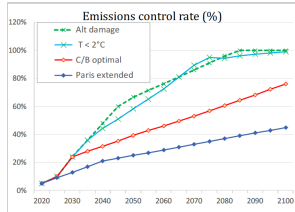
Figure 4. Emissions control rate for CO_2 and abatable GHGs (percent of no control)

1. New optimal path calls for higher control rates/SCC than old. We are starting further behind.

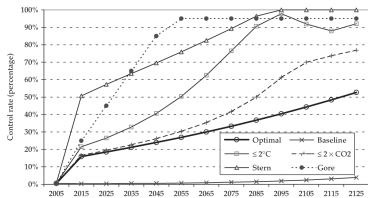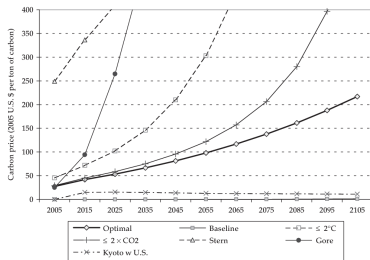**Figure 5-4.** Carbon prices under different policies. The globally averaged carbon price under different policies over the next century. Note the upward tilt of the strategies. These prices are per ton of carbon; for prices per ton of CO_2, divide by 3.67.

Table 5. Price of CO_2 emissions (2019 $/tCO_2$)
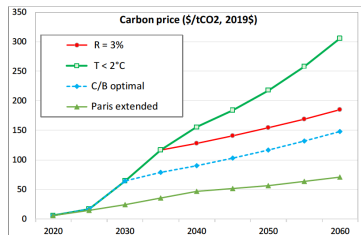
Figure 5. Price of CO_2 emissions (2019 $/tCO_2$)

1. New is CO_2, old is C, so multiply new by 44/12.
2. Old: 2030 has 60$ t, New: 2030 has 60$t CO_2. 44/12 as big.
3. New optimal path calls for higher control rates/SCC than old. We are starting further behind.
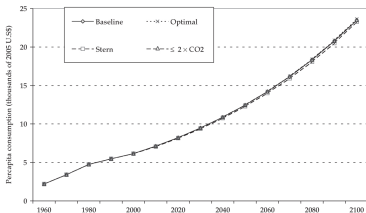
Figure 5-9. Per capita consumption, major runs. The trend of per capita consumption is strongly rising in the DICE-2007 model projections. Also, the levels of consumption are virtually indistinguishable among the different policies.
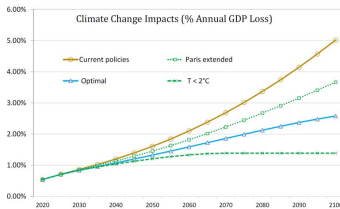
Figure 6. Climate change impacts (annual percent of GDP loss)

1. These pictures are not as different as they look. Old is levels, New is ratios. A bad climate change policy is still about as important as a bad recession.

Tipping points I

A really interesting recent paper (Dietz et al. PNAS 2021) looks at what happens to the social cost of carbon if we allow for 'tipping points' in the carbon cycle or in damages.

They consider tipping points already considered in another paper/IAM,

Table 1. Models synthesized in this study

| Tipping point	Papers	IAM	Model of TP	Uncertainty
Permafrost carbon feedback (PCF)	Hope and Schaefer (24)	PAGE09	Process based	MC
	Kessler (25)	DICE	Process based	Deterministic and MC
	Yumashev et al. (23)	PAGE-ICE	Process based	MC
Ocean methane hydrates (OMH)	Ceronsky et al. (50)	FUND	Tipping event	Deterministic and MC
	Whiteman et al. (51)	PAGE09	Tipping event	MC
Arctic sea ice/Surface Albedo Feedback (SAF)	Yumashev et al. (23)	PAGE-ICE	Process based	MC
Amazon dieback (AMAZ)	Cai et al. (14)	DSICE	Tipping event	Survival analysis
GIS disintegration	Nordhaus (19)	DICE	Process based	Deterministic
WAIS disintegration	Diaz and Keller (47)	DICE	Tipping event	Survival analysis
Atlantic Meridional Overturning Circulation (AMOC) slowdown	Anthoff et al. (22)	FUND	Tipping event	Deterministic
Indian summer monsoon (ISM) variability	Belaia (48) using Schewe and Levermann (52)	RICE	Process based	Stochastic

MC, Monte Carlo simulation.

- GIS= Greenland Ice Sheet, WAIS= West Antarctic Ice sheet.

Tipping points II

- Idea: Evaluate all tipping points that people have considered in using the same IAM. The exact equations look similar to DICE, but are complicated by the tipping points (see their appendix).

- It works by supposing that a tipping point arrives at a random time, and then calculates the average change in SCC that this implies in many trials.

Tipping points III

Here is what they find,

Table 2. The SCC (2020 US dollars) and the percentage change in the SCC due to tipping points collectively and individually

TP	Expected SCC, US$/tCO$_2$	Increase due to TP, %
None	52.03	—
Permafrost carbon	56.41	8.4
Ocean methane hydrates	58.85	13.1
SAF	51.14	−1.7
Amazon	52.07	0.1
GIS	52.97	1.8
WAIS	53.57	2.9
AMOC	51.28	−1.4
Indian summer monsoon	52.70	1.3
All TPs	64.80	24.5
∑ main effects, all TPs	—	24.5
All costly TPs	67.05	28.9
∑ main effects, costly TPs only	—	27.6

The expected SCC is computed over 10,000 Monte Carlo draws with 0.1% trimmed. Specification comprises RCP4.5-SSP2 emissions and GDP/population growth, Hope and Schaefer PCF, Whiteman et al. beta OMH, and IPSL AMOC hosing. TP, tipping point.

- Adding tipping points does not have a big implication for policy. It bumps up the optimal carbon tax from about 50 to about 75$ per ton of CO_2 .
- This is very surprising.

Tipping points IV

- Why? I think it is because all of these tipping points happen a long way in the future, and with discounting (they use about 5%) this means they are not very important.

Pindyck Critique

Pindyck makes two critiques of the economic analysis of climate change. They are mainly directed at the literature based on IAMs, which builds on Nordhaus and is a lot more complex.

- That uncertainty is so important that precise estimates of the social cost of carbon are misleading. We should not expect more precision that we would expect for the choice of defense spending.

- That uncertainty is a fundamental part of the problem and means that we should be thinking about policy as insurance against catastrophes rather than moving the mean.

Distribution of Climate Sensitivity

Histogram of "Best Estimates" from Climate Sensitivity Studies

☐ Studies published 2010 onwards ■ Studies published before 2010
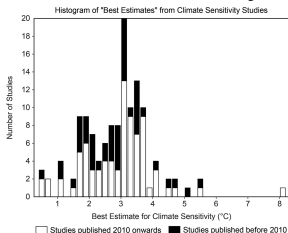
- Histogram of climate sensitivities drawn from 131 studies. Black is pre2010. White is post. Post-2010 density is flatter, so uncertainty is increasing. Mean/s.e. (<2010)= $2.77(1.03)$; (>2010) = $2.87(1.11)$.
- Time frame for climate sensitivities is not specified. Are these fair comparisons? We don't want to compare 20 year and 100 year climate sensitivities.
- Implicitly, this figure imagines that each study is a draw from the true distribution of climate sensitivities. Does this seem right?

Insurance I

- Climate uncertainty together with a convex damage function creates a role for 'insurance'.

- Nordhaus damage function, share of GDP lost at ΔT, is

$$L(\Delta T) = 1 - \frac{1}{1 - 0.0045\Delta T + 0.0035\Delta T^2}$$

Pindyck uses something a little simpler

$$L_P(\Delta T) = 1 - \frac{1}{1 + 0.01\Delta T^2}$$

Insurance II

- So we have

$$L_P(0) = 0$$
$$L_P(2) = 0.04$$
$$L_P(4) = 0.14$$

- Consider two lotteries,
  - 2°warming for certain, so $L_p = 0.04$
  - Equal chance of 0 or 4°of warming, so
    $E(L_p) = \frac{1}{2}L_P(0) + \frac{1}{2}L_P(4) = 0.07$

  so we would pay to avoid uncertainty.

- This is the sense in which climate change policy is 'insurance'.

- This issue is compounded by the fact that the utility function is also concave.

Insurance III

If you don't like Nordhaus' conclusion, 'climate change is a bad recession', Pindyck is trying to give you an out. There is too much uncertainty for this sort of precision, and the uncertainty is itself costly and something we want to avoid.

Nordhaus' rejoinder I

Chapter 7 of Nordhaus' book is all about uncertainty.

How does it work? For each important variable, Nordhaus looks at the mean and s.e. of his estimate, and asks how the SCC would change for a k standard deviation change in that parameter. This lets him trace out Pindyck's L function for each model parameter.

Nordhaus' rejoinder II

Here is the table where he describes parameters and uncertainty,

Table 7-1. Major Assumptions about Uncertain Parameters in Uncertainty Runs

Variable	Definition	Units	Mean	Standard Deviation
g(TFP)	Rate of growth of total factor productivity	Per year	0.0092	0.0040
g(CO_2/GDP)	Rate of decarbonization	Per year	−0.007	0.002
$T_2 \times CO_2$	Equilibrium temperature-sensitivity coefficient	°C per CO_2 doubling	3.00	1.11
DamCoeff	Damage parameter (intercept of damage equation)	Fraction of global output	0.0028	0.0013
P(back)	Price of backstop technology	$ per ton of carbon replaced	1,170	468
Pop	Asymptotic global population	Millions	8,600	1,892
CarCyc	Transfer coefficient in carbon cycle	Per decade	0.189	0.017
Fosslim	Total resources of fossil fuels	Billions of tons of carbon	6,000	1,200

Note: The mean values and standard deviations of the uncertain parameters used in this chapter. For a detailed discussion of the derivation of the parameters, see "Accompanying Notes and Documentation of DICE-2007 Model" (Nordhaus 2007a).

Focus on climate sensitivity, $T_2 \times CO_2$. The mean and standard error match Pindyck closely.

Nordhaus' rejoinder III

Here is what he finds.

Table 7-2. Uncertainty Results for the Social Cost of Carbon, 2005

| | Value of SCC for different uncertain parameters | | | | | | | | Prob (x > x*) | |
| | (2005 $ per ton of carbon in 2005) | | | | | | | | | |
Sigma	g(TFP)	g(CO₂/GDP)	T2xCO₂	DamCoeff	P(back)	Pop	CarCyc	Fosslim	Normal	t(5)
0	28.10	28.10	28.10	28.10	28.10	28.10	28.10	28.10	0.5000	0.5000
1	36.07	28.27	38.07	40.99	28.10	32.14	29.16	28.10	0.1587	0.2047
2	48.08	28.43	46.44	53.89	28.10	35.91	30.32	28.10	0.0228	0.0579
3	51.21	28.60	53.49	66.80	28.10	39.44	31.61	28.10	0.0013	0.0169
4	54.68	28.76	59.47	79.73	28.10	42.75	33.04	28.10	3.17 E-05	0.0057
5	58.52	28.92	64.59	92.66	28.10	45.84	34.62	28.10	2.87 E-07	0.0022
6	62.80	29.09	69.03	105.61	28.11	48.75	36.39	28.10	9.87 E-10	0.0010

Note: The value of the social cost of carbon is shown for the mean values of the parameters and for the mean plus sigma times the number of standard deviations in the "sigma" column. Each column shows the results from varying only the listed parameter while holding all other parameters at their mean value. We have varied the parameter in the direction in which the social cost of carbon increases. For example, if the damage coefficient is one standard deviation above its mean, then the social cost of carbon is $40.99 per ton of carbon rather than $28.10 per ton of carbon at its mean value.

Variable key:

Sigma =number of standard deviations from the mean; g(TFP) = growth in total factor productivity; g(CO₂/GDP) = rate of decarbonization; T2×CO₂ = temperature-sensitivity coefficient; DamCoeff = intercept of damage function; P(back) = price of backstop technology; Pop = asymptotic population; CarCyc = atmospheric fraction in carbon cycle; Fosslim = resource abundance of carbon fuels; P(x >x*) = probability that value will exceed the value at that level of sigma for normal and Student's t distribution with 5 degrees of freedom

- 'Sigma' is # of standard deviations.

Nordhaus' rejoinder IV

- Normal is probability of Sigma $\times$ s.e. if parameter is distributed Normal with given mean and standard deviation, i.e. $\Phi(\text{mean, s.e}^2)$. t(5) is the same for a t distribution (with a little bit fatter tails.

- Extreme draws of parameters don't affect the SCC very much. In particular, even if we drew $\rho = 3 + 4 \times 1.11 = 7$ the SCC in 2005 is just 60.

- Compare this to e.g., the Stern plan, where it is about 250\$.

- Thus, Nordhaus' analysis suggests that the slope of the Pindyck's *L* function is pretty flat. Damages don't increase very rapidly with climate sensitivity. We can get a really bad draw of climate sensitivity and it won't affect policy very much.

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Nordhaus' rejoinder V

- This seems surprising, too. What is going on? I don't know. My guess is that it is discounting again.

A simple IAM I

BDICE is too simple to be useful for making quantitative predictions. DICE is a little too complicated to experiment with. Here is a model that about splits the difference. You can use it to experiment with different parameter values and policy experiments.

A simple IAM II

Here are the main equations;

$$min_{\mu_t} \sum_{t=2000}^{2500} \frac{D(T_t)Y_t + TAC(\mu_t)}{(1+r_t)^{t-2000}}$$

$$s.t. E_t = (1-\mu_t)\sigma_t Y_t$$

$$P_t = P_{2000} + \chi \sum_{t=2000}^{2500} E_t$$

$$T_t = T_{2000} + \eta \ln (P_t / P_{2000})$$

$$D(T_t) = \phi_1 T_t^{\xi_2} Y_t$$

$$TAC_t = \theta_1 (\mu_t)^{\theta_2} Y_t$$

$$r_t = \rho + \alpha g_t$$

A simple IAM III

$P \sim$ ppm CO_2 . T, E, $Y \sim$ climate, emissions, world GDP. $D \sim$ climate damage as a share of income. $\mu \sim$ mitigation rate. $TAC \sim$ total abatement cost as share of income.

- Cost minimization rather than utility maximization.
- η is climate sensitivity (more later)
- D is climate damage as a share of income.
- TAC is mitigation cost as a share of income.
- Interest follows the Ramsey rule we've already talked about.
- Evolution of Y is not modelled.

A simple IAM IV

You can evaluate this model in a spreadsheet.

Your homework asks you to experiment a little bit with this.

Conclusion

- We've now evaluated the DICE model, as promised.
- The conclusion is surprising – climate change is about as important as a big recession.
- This reflects discounting. Damages come late, abatement costs come early.
- This result seems to be robust to uncertainty.
- This result seems to be robust to various tipping points.
- You can experiment with the 'simple IAM' and see if you can break this result.
- The only ways (so far) to really change this result is with a low interest rate that (I think) is hard to defend. Allowing climate to retard economic growth will also do it. Though there is not much evidence for this.
- What would Oreske and Owens say? Do you believe it?